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# The Impact of Illiquidity on the Asset Management of Insurance Companies<sup>1</sup>

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# The Impact of Illiquidity on the Asset Management of Insurance Companies

## Abstract

This paper investigates the impact of illiquidity on insurance company asset allocation and selling strategy. The need for insurers to settle claims as they arise by freeing funds immediately make insurers especially susceptible to the effects of transaction illiquidity. Using a simplified model of a risk neutral insurance company, we examine the effect of permanent and temporary price impact on initial asset allocation. The optimal asset allocation and selling strategy are determined numerically. While a clear diversification benefit is evident on the basis of illiquidity, under certain market assumptions, the cash-flow matching strategy is optimal.

**Key words:** Liquidity, Price Impact, Optimal Liquidation, Asset Liability Management, Portfolio Diversification

# 1 Introduction

Insurance companies are some of the largest investors in world financial markets. While there are many similarities between an insurance company and any other large institutional investor, it is worthwhile to examine some of the differences. Insurance companies collect premiums and then payout claims as they arise. Some types of insurance tend to have a relatively predictable claim distribution, automobile insurance, others are much more variable, e.g. industrial fire insurance and hurricane insurance. Even automobile insurance can suddenly require very large claim payments, for example, a bad winter storm in the Northeastern US seaboard. Investment strategies for insurance companies must allow for providing cash at irregular and unpredictable intervals. This issue is not only important when selling investments, but must be considered at the time of the initial allocation.

When modelling the interaction between an insurance company and markets, it is important to consider these constraints. For example, modern finance theory builds on the competitive market paradigm (see, e.g. Duffie (1992)). The competitive market paradigm depends on the implicit assumption that security markets are perfectly elastic. Thus investors are price takers who can buy and sell unlimited amounts of securities without changing their price.

In reality, however, the trades of large institutional investors, such as insurance companies, dealers, mutual funds, and pension funds, have a significant price impact. There are many empirical studies supporting this view. For example, Holthausen, Leftwich, and Mayers (1990) examine the 50 largest buy and sell trades in 1983 for 109 randomly selected NYSE-listed firms. They find a price impact of around 1 percent. Breen, Hodrick, and Korajczyk (2002) examine NYSE and AMEX-listed stocks from 1993 to 1997. They determined that an increase of net turnover during a five-minute interval by 0.1%

of the shares outstanding produces an average price impact of 2.65%. Keim and Madhavan (1996) examine block trades of small NYSE, AMEX and NASDAQ-listed firms from 1985 to 1992. They calculated a much larger price impact of around 8%. These and other similar results suggest that the price for trading one unit of a security is different from the price for large transactions, or in other words: there is a "quantity" effect on price.<sup>1</sup>

The market microstructure literature explains such deviations from perfectly liquid market conditions, analyzing the dynamic strategies of individual market participants. These analysis are based on assumptions about the market participants' trading motives. Trading motives are generally divided into "informational" motives due to private information about asset payoffs, and "allocational" motives, such as portfolio rebalancing, risk-sharing and the need for cash.<sup>2</sup> There exists a broad literature on dynamic strategies of traders with informational motives. One of the most prominent examples of this strand of literature is Kyle (1985). In his model, a risk-neutral trader with insider information trades with risk-neutral market makers. The market makers cannot differentiate between the insider and noise traders also present in the market, and thus agree to trade with the insider. Kyle shows that it is optimal for the insider to trade slowly, and hence reveal his information slowly, until it is publicly announced. Since the trade itself contain information, the market makers watch the order flow and adjust prices accordingly. This mechanism creates price impact.<sup>3</sup>

The dynamic strategies of traders with allocational motives have attracted comparably less academic attention. Vayanos (2001) studies a large, risk averse trader who receives a risky stock endowment every period, and trades with risk averse market makers to share risk. The market makers cannot distinguish the large trader from small noise traders who are also present in the market. Vayanos shows that after an endow-

ment shock, there are two optimal strategies for the large trader to reduce his stock holdings to the long-run limit: selling stocks at a decreasing rate, or selling more than the long-run limit and buying back the difference, when the price falls. This second strategy can be seen as "market manipulation". The large trader knows that he will sell, he knows how many shares he will sell and he knows that the price will fall. If this information is not reflected in the price, the large trader has an incentive to "frontrun" on his then private information. In this model, price impact follows from the assumption that market makers are risk averse. They simply have to be compensated for taking over large risky positions.

These microstructure models of illiquid markets are in general relatively simple, while solving for an equilibrium price in a realistic and dynamic model is very complex. Therefore existing market microstructure models provide an important conceptual background, but find little concrete applications in asset management procedures. The purpose of this paper is to analyze the impact of illiquidity in security markets on the asset management of property and casualty insurance companies. The trading behavior of insurance companies is determined by its insurance business. Up front premium payments have to be invested in financial assets, and in the event of larger claims, these assets have to be sold again to settle the claims. Clearly an insurance company's trading motives are allocational ones.

There is little literature on the dynamic behaviour of large traders with allocational motives from a partial equilibrium perspective. Bertsimas and Lo (1998), Almgren and Chriss (2000), Huberman and Stanzl (2000) and He and Mamaysky (2001) study the dynamic strategies of large traders who have to complete a trade within a fixed time horizon, facing an exogenously given price impact function. We build on this framework for our analysis. Since these models examine only one block trade, we have to choose

whether to focus on buy or sell transactions. One of the unique characteristics of property and casualty insurance companies is their stochastic demand for large amounts of cash in the event of big claims. Therefore, the liquidation of financial assets is frequently necessary to cover all claims. We hence choose to focus on the liquidation side of the asset management process. In addition to our analysis of liquidation strategies, we examine the influence of illiquidity on the asset allocation decisions of property and casualty insurance companies.

Surprisingly, there has been little published work on the influence of illiquidity on trading strategies and asset management decisions for the insurance industry. Berry-Stölzle (2005) compares selected selling strategies for a property and casualty insurance company in a market with stochastic spreads. However, the idea of combining the dynamic liquidation models with the collective model of risk theory to develop a liquidation model of a whole insurance company is promising.<sup>4</sup> We apply this basic concept in designing our model.

To clarify, the purpose of our paper is not to explain the appearance of price impact or other forms of market illiquidity, but to study optimal asset management decisions of property and casualty insurance companies in an illiquid securities market. Consequently, this paper takes a partial equilibrium perspective. We propose a cash-flow based liquidation model of an insurance company and analyze selling strategies and asset allocation decisions for a portfolio with cash and an illiquid asset. Within this framework, we study the influence of price impact on the expected surplus and determine an solution set consisting of an optimal initial asset allocation and an optimal liquidation strategy. We show that the initial asset allocation, in conjunction with the appropriate liquidation strategy, is an important tool in maximizing shareholder value.

In our model, we assume that all premium payments have already been received at

the beginning of the period under consideration and have been invested in a mix of two different assets: cash and a risky stock. Therefore, the insurance company has to sell securities for the settlement of insurance claims. We incorporate market illiquidity in our model by assuming that the insurance company has to make a price concession when selling securities. Thus the securities are illiquid in the sense that trading moves their prices. We distinguish between two types of price impacts: temporary and permanent. Permanent price impact results from a shift in the equilibrium price of the stock under consideration due to trading activity (see, e.g. Kyle (1985)). Following the empirical work of Kraus and Stoll (1972), and the subsequent work of Holthausen, Leftwich, and Mayers (1987), Holthausen, Leftwich, and Mayers (1990), Chan and Lakonishok (1993) and Chan and Lakonishok (1995), we assume that in addition to the permanent price impact, there is a temporary one. Temporary price impact results from temporary imbalances in supply and demand caused by trading. It can be interpreted as transaction costs applicable to single trades.

Focusing on the influence of illiquidity on asset management decisions of insurance companies, we assume that the insurance company is risk neutral, and optimizes its expected terminal surplus. We examine only liquidation strategies and exclude any reinvestment. These restrictions eliminate the problem of market manipulation being an optimal strategy as in Vayanos (2001). We further assume that regulatory requirements do not allow short-sales. Concerning financial leverage, we impose a rather weak restriction. The insurance company has to assure that there is no expected financial leverage. This weak restriction allows us to perform our analysis solely from the aspect of expected value. Abstracting from market risk considerations, the influence of illiquidity on the asset management can be examined more clearly.

Our analysis is divided into three sections. First, we study the influence of tempo-

rary and permanent price impact on the optimal liquidation strategy for given initial asset allocations. Second, we examine the influence of price impact on the maximum expected surplus and the optimal solution set consisting of a liquidation strategy and an initial asset allocation. Third, we discuss the implications of our results for the asset management process of insurance companies.

Our analysis provides three main results: First, the optimal asset allocation converges to a limit for increasing price impact. In the limit, it is optimal to hold the expected value of the insurance claims in cash and invest the spare money in the stock market. Since our analysis is solely from the aspect of expected value, the insurance company can pay all expected claims with cash and does not have to liquidate stocks. This behavior can be interpreted as the utilization of a cash-flow matching strategy. Second, the influence of permanent price impact on optimal liquidation strategies and asset allocation decisions is much stronger than the influence of temporary price impact. Permanent price impact adversely influences the future stock price. Ignoring commonalities of stock returns, a negative future stock price development only influences an insurance company's portfolio value, when the company still holds a position of this stock. Therefore, it is advantageous for the insurance company to liquidate a stock position in one block sale. This means that there is an incentive for the insurance company to require each individual asset position in its portfolio to be small enough for a single block sale. Summing up, there is a diversification benefit in illiquid markets apart from the one due to market risk considerations introduced by Markowitz (1952). This is our third result.

The remainder of this paper is organized as follows. In Section 2 we present the model. In Section 3 we analyze optimal liquidation strategies and optimal asset allocation decisions for the model insurance company under various parameter settings. In addition to the empirical results, Section 3 also describes the optimization procedure.

Section 4 discusses the implications of our results for the asset management of insurance companies. The final section concludes and puts forth additional research topics.

## 2 Model

In this section, we introduce the theoretical foundation for our model. This model is a selective simplification of an insurance company as a pure liquidation enterprise. The focus lies on cash-flows only.

For this model we make a number of simplifying assumptions. The time of consideration is a one year period. We assume that the company underwrites the same amount and type of insurance business every year; in other words, the insurance business stays constant over time. Under this assumption the (expected) claim payments within the one year under consideration for business written in this year as well as in previous years approximately equals the (expected) payments for claims from this year's business, which may be settled the same year or in future years. Thus, if we ignore the time value of money, we can substitute the claim payments within the period under consideration for the claim payments of the business written in this period.<sup>5</sup>

Considering the claim payments, we assume that the sequence of inter-occurrence times  $\{T_i, i \geq 1\}$  consists of independent random variables with an exponential distribution  $Exp(\lambda), \lambda > 0$ . The claim sizes  $\{U_i, i \geq 1\}$  are independent and identically distributed and independent of the sequence  $\{T_i\}$  of inter-occurrence times. These assumptions lead to the classical collective risk model of actuarial risk theory, where the cumulative claim amount  $\{Y_t, t \geq 0\}$  follows a compound Poisson process. Using the notation  $\vartheta_i = \sum_{k=1}^i T_k$  for the points of time at which claim payments occur, the aggregated

claim amount in the interval  $(0, t]$  can be expressed as

$$Y_t = Y(t) = \sum_{i=1}^{\infty} U_i \mathbb{I}[\vartheta_i \leq t] = \sum_{i=1}^{N(t)} U_i, \quad (1)$$

where  $\mathbb{I}(\cdot)$  is the indicator function and  $\{N(t), t \geq 0\}$  the counting process given by

$$N(t) = \sum_{i=1}^{\infty} \mathbb{I}(\vartheta_i \leq t). \quad (2)$$

We further assume that all premium payments have been received at the beginning of the year and are already invested in financial assets. Therefore securities have to be sold for the settlement of insurance claims. Assuming that the execution of trades is only possible at certain points of time, we face a discrete model framework. Formally, the one year period under consideration is divided into  $T$  subperiods  $(t-1, t], t = 1, \dots, T$  and trading can take place at points of time  $t = 1, \dots, T$ . Using the compound Poisson process from equation (1), the aggregate claim amount  $C_t$  of period  $t = 1, \dots, T$  is given by

$$C_t = Y(t) - Y(t-1). \quad (3)$$

At the beginning of the year, the insurance company is invested in a mix of two different assets: cash and a risky stock. At points of time  $t = 1, \dots, T$  the insurance company has to sell financial assets to pay off the claims  $C_t$  and can rebalance its portfolio concurrently. Let  $M_t$  denote the number of units of cash the insurance company is holding at time  $t$ , and let  $N_t$  denote the number of units of the risky stock the insurance company is holding at time  $t$ . The starting values  $(M_0, N_0)$  represent the initial asset allocation of the insurance company. Since this paper focuses on the liquidation part of the asset management process, we will refer to the set  $(M_t, N_t), t = 1, \dots, T$  as the insurance company's liquidation strategy.

Cash earns a constant rate of return  $r$  each period. Therefore the value of the cash position  $B_t$  follows the discrete process

$$B_{t+1} - B_t = rB_t. \quad (4)$$

This corresponds to an Euler discretization of the commonly used process  $dB_t = rB_t dt$ . The price of a perfectly liquid stock is given by

$$S_{t+1} - S_t = S_t(\mu + \sigma\varepsilon_{t+1}). \quad (5)$$

Assuming that the random shock  $\varepsilon_{t+1}$  is distributed  $\varepsilon_{t+1}|t \sim N(0, 1)$  at time  $t$ , but known at time  $t + 1$ , this process is an Euler discretization of the geometric Brownian motion  $dS_t/S_t = \mu dt + \sigma dZ_t$ .

We now leave the perfectly liquid world of the competitive market paradigm which builds the basis of modern finance theory (see Duffie (1992), Jarrow and Turnbull (1996)). By introducing price impact, we acknowledge that the insurance company has to make a price concession when selling the stock. Hence, the stock is illiquid in the sense that trading moves its price. Following the work of Kraus and Stoll (1972), and the subsequent work of Holthausen, Leftwich, and Mayers (1987), Holthausen, Leftwich, and Mayers (1990), Chan and Lakonishok (1993) and Chan and Lakonishok (1995) we distinguish between two types of price impact: temporary and permanent (see Figure 1). Temporary price impact is caused by temporary imbalance in supply and demand caused by our trading. It can be interpreted as transaction costs applicable to single trades. Permanent impact, however, results from a shift in the equilibrium price of the stock under consideration due to trading activity. This form of price impact can be motivated theoretically by the existence of a market maker extracting information from

the order flow and adjusting prices accordingly (see, e.g. Kyle (1985)).

- Please insert FIGURE 1 approximately here -

Following Pereira and Zhang (2004), we consider a permanent price impact function similar to He and Mamaysky (2001) and Breen, Hodrick, and Korajczyk (2002). Abstracting from any other influences, the price change caused by trading is given by

$$(S_{t+1} - S_t)/S_t = \hat{\psi} \frac{N_{t+1} - N_t}{F} \quad (6)$$

where  $S_t$  is the stock price at time  $t$ ,  $N_{t+1} - N_t$  specifies the number of shares the insurance company trades,  $F$  represents the float of the stock, and  $\hat{\psi}$  is a positive parameter. Thus, the permanent price impact is proportional to turnover. This assumption is in line with Huberman and Stanzl (2004), who proved that only linear price impact functions rule out quasi-arbitrage and hence support feasible market prices.<sup>6</sup> Note that  $N_{t+1} - N_t$  is negative for selling transactions and positive for buying transactions. In other words the insurance company receives a lower price if it sells the stock, and pays a higher price if it buys the stock.

In our world with price impact, the price of an illiquid stock is influenced by three factors: drift, volatility, and price impact. We therefore define the stock price process  $S_t$  as the sum of the two components (5) and (6):

$$S_{t+1} - S_t = S_t[\mu + \sigma\varepsilon_{t+1} + \psi(N_{t+1} - N_t)] \quad (7)$$

renaming  $\psi \equiv \hat{\psi}/F$  for simplicity.

So far, the model of the asset prices is identical to the one used by Pereira and Zhang (2004). However, in addition to the permanent price impact the insurance company in our model also faces temporary price impact. When the insurer sells a large number of shares, the execution price may decrease, partially due to bid-orders being executed faster than new orders arrive. Therefore the company has to make a concession on the price. In other words, the insurer faces temporary transaction costs that will be offset in the next period when new orders have arrived. We introduce the temporary price impact function

$$h(N_{t+1} - N_t) = \hat{\phi} \frac{N_{t+1} - N_t}{F} \quad (8)$$

quantifying the temporary drop in price per share caused by trading  $(N_{t+1} - N_t)$ -shares. The parameter  $F$  in this equation represents the float of the stock, and  $\hat{\phi}$  is a positive constant. Then the actual transaction price per share at time  $t + 1$  is given by

$$\tilde{S}_{t+1} = S_{t+1} + \phi(N_{t+1} - N_t), \quad (9)$$

where we have redefined  $\phi \equiv \hat{\phi}/F$  for convenience. Note that  $N_{t+1} < N_t$  for sales and hence the actual price per share received on the sale is smaller than the stock price update  $S_{t+1}$ . For buying stocks, however, the execution price is higher than the stock price update.

The sequence of events in our model is as follows (see figure 2). At time  $t + 1$  the insurance company observes the claim amount  $C_{t+1}$  and the pre-transaction price  $\bar{S}_{t+1} \equiv S_t[1 + \mu + \sigma\varepsilon_{t+1}]$ . As both the permanent and the temporary price impact are deterministic functions of trading activity, the insurance company can incorporate this information into its liquidation decision. This model setup assumes that the insurance company is a well informed investor with access to the whole order book of the stock exchange.<sup>7</sup> Seeing the whole demand-supply schedules and anticipating the market

resiliency, the insurer chooses  $N_{t+1}$ . The sell order of  $N_{t+1} - N_t$  shares then clears the bid side of the order book up to a certain point. In other words: the trade is executed, and the transaction price is recorded. Abstracting from details of the market microstructure, we assume that there is one transaction price  $\tilde{S}_{t+1}$  for the entire order. Other market participants observe the transaction and adjust their assumptions about the value of the stock according to the information contained in the trade. New orders arrive and fill the order book again, creating the new midpoint of the bid-ask:  $S_{t+1}$ . This mechanism offsetting the temporary price impact is called resiliency and is specific for each combination of market and stock.

- Please insert FIGURE 2 approximately here -

We require the insurance companies portfolio strategy to be self-financing and thus fulfilling the following equation:

$$M_t B_{t+1} + N_t S_{t+1} = M_{t+1} B_{t+1} + N_{t+1} S_{t+1} + C_{t+1} + \phi |N_{t+1} - N_t| \quad (10)$$

This means that the insurance company's current expenses for claims and trading as well as its current portfolio of financial assets, has to be self-financed by its asset management activity up to now. Since  $N_{t+1} - N_t$  can be positive or negative, we have to consider the absolute value to include the temporary trading costs with a positive sign. The permanent price impact of trading is already contained in the stock price  $S_{t+1}$  and does not have to be considered explicitly.

Defining the market value of the insurance company's assets at time  $t$  as  $A_t \equiv M_t B_t + N_t S_t$ , and using equation (10), the change in the insurer's asset value can be

expressed as  $A_{t+1} - A_t = M_t(B_{t+1} - B_t) + N_t(S_{t+1} - S_t) - C_{t+1} - \phi|N_{t+1} - N_t|$ . We then substitute  $M_t = (A_t - N_t S_t)/B_t$  and use equations (4) and (7) to arrive at

$$A_{t+1} = A_t(1 + r) + N_t S_t[\mu - r + \sigma \varepsilon_{t+1} + \psi(N_{t+1} - N_t)] - C_{t+1} - \phi|N_{t+1} - N_t|. \quad (11)$$

The purpose of this paper is to analyze the influence of illiquid securities markets on the asset management of insurance companies. To avoid any influence of market risk in our results, we assume that the insurance company is risk neutral, and optimizes the expected value of its terminal assets. Since all insurance claims are already paid at time  $T$ , the expected value of the insurance company's terminal assets is equivalent to its expected surplus.

This model focuses on two aspects of the asset management process: the initial asset allocation and optimal liquidation strategies. We therefore require the stock holdings  $N_{t+1}$  at time  $t + 1$  to be smaller or equal than the stock holdings  $N_t$  at time  $t$ . This means that reinvestments in stocks are not possible within the model setup.

We assume that regulatory requirements do not allow short-sales, so  $N_t \geq 0$ . Concerning financial leverage, we impose a rather weak restriction. The insurance company has to assure that there is no expected financial leverage. However, in a lot of scenarios the company can still borrow money for paying off the claims and stay invested in the stock market. This weak restriction allows us to perform our analysis solely from the aspect of expected value. Abstracting from risk considerations, the influence of illiquidity on the asset management can be examined more clearly.

Summing up, we formulate the insurance company's optimization problem:

$$\Theta(A_0) \left\{ \begin{array}{l} \mathbb{E}[A_T] \rightarrow \max \\ A_{t+1} = A_t(1+r) + N_t S_t [\mu - r + \sigma \varepsilon_{t+1} + \psi(N_{t+1} - N_t)] - C_{t+1} - \phi |N_{t+1} - N_t| \\ S_{t+1} - S_t = S_t [\mu + \sigma \varepsilon_{t+1} + \psi(N_{t+1} - N_t)] \\ N_{t+1} \leq N_t \quad t = 0, 1, \dots, T-1 \\ N_t \geq 0 \quad t = 0, 1, \dots, T \\ N_t = 0 \quad \{t = 0, 1, \dots, T | \mathbb{E}[A_t] - N_t \mathbb{E}[S_t] < 0\} \end{array} \right.$$

We optimize over the sequence  $\{N_t\}_{t=0}^T$ . The notation  $\Theta(A_0)$  highlights the fact that this is an optimization program given the market value  $A_0$  of the insurance company's assets at time 0. Since  $M_t = (A_t - N_t S_t)/B_t$ , the optimization gives us the optimal liquidation strategy  $(M_t, N_t), t = 1, \dots, T$  as well as the optimal initial asset allocation  $(M_0, N_0)$ .

### 3 Numerical Results

In our numerical analysis, we calibrate the model to empirically reasonable parameter values and examine the impact of different levels of illiquidity on the insurance company's optimal liquidation strategy as well as the optimal initial asset allocation.

For our baseline case, we use a property and casualty insurance company with an industrial fire insurance business line. In his actuarial text book Mack (2002), Mack estimates parameters for different claim size distributions using a sample data set from industrial fire insurance (see p. 87 ff.). Following this example, we suppose that the claim size measured in US-\$ 1,000 follows a lognormal distribution  $U_i \sim LN(a, b)$  with a scale

parameter  $a = 1.61$  and a shape parameter  $b = 1.96$ . In our model the inter-occurrence times between claims measured in months are exponentially distributed  $T_i \sim Exp(\lambda)$ . We use the parameter value  $\lambda = 0.0042$ . This implies that in the long run there are an average of 238 claims per month or 2,857 per year.

At time  $t = 0$ , we endow the insurance company with financial assets worth 150 mio. US-\$ ( $A_0 = 150,000,000$ ). This amount is high enough to ensure that the insurance company can pay their liabilities more than 99.5% of the time.<sup>8</sup>

The expected stock return  $\mu$  in equation (7) does not include any liquidity premium. We therefore calibrate the expected return  $\mu$  and the standard deviation  $\sigma$  of the stock price process to those of very liquid stocks. Following Pereira and Zhang (2004), we use the largest (top decile) New York Stock Exchange listed stocks for the calibration. A portfolio of these stocks had an average annual return of 11% and an average annual volatility of 18% from 1926 to 2002. We assume that the insurance company settles claims monthly, and hence liquidates financial assets on a monthly basis. So, we set  $\mu = 0.11/12$  and  $\sigma = 0.18/\sqrt{12}$ . The interest rate is set to  $r = 0.05/12$ .

Concerning the price impact parameters  $\psi$  and  $\phi$ , we will use a wide range of values, and study their impact on asset management decisions of an insurance company. However, the empirical results of Hausman, Lo, and MacKinlay (1992), Kempf and Korn (1999), and Brennan and Subrahmanyam (1996) indicate that it is realistic to assume an overall price impact parameter of  $10^{-5}$  for large transactions in liquid stocks and price impact parameters up to  $10^{-3}$  for illiquid stocks.

### 3.1 The Optimization Procedure

We are looking for the sequence of stock holdings  $N_t, t = 0, 1, \dots, 12$  that maximizes the expected value of the insurance company's terminal asset holdings  $E[A_T]$ , subject

to some constraints. We compute this expected value through Monte Carlo simulations, using 100,000 scenarios.

To improve the accuracy and reduce cpu-time, we split the overall optimization into two separate loops. In the inner loop the optimal liquidation strategy  $N_t, t = 1, \dots, 12$  is determined, given the initial asset allocation  $N_0$ . In the outer loop we then optimize the initial asset allocation.<sup>9</sup>

For solving the inner loop, we first perform a grid search, providing us with possible starting points for an optimization routine. We then use the downhill simplex method according to Nelder and Mead (1965) in combination with penalty cost functions to pin down the solution.<sup>10</sup> The multidimensional downhill simplex method has one big advantage, it requires only function evaluations and no derivatives. This property makes it very stable and suitable for optimizing a simulation procedure in twelve dimensions. The outer loop is solved through a combination of grid search and Golden Section search with parabolic interpolation, which is a stable algorithm as well.<sup>11</sup>

The optimization procedure we implement here is a static and not a dynamic one. Undoubtedly, a dynamic programming algorithm has a theoretical advantage. Its solution consists of a sequence of functions mapping all possible states of the system to the possible actions. This means that a decision maker can make decisions at every point of time  $t = 1, \dots, T$ . However, in the static procedure, the decision maker chooses the optimal strategy at the beginning of the period under consideration and cannot change this strategy thereafter. Therefore the decision maker cannot use information about the state of the system that will be available at future points of time. We argue that there is not much additional information arriving in the future and that the difference to a dynamic programming solution is negligibly small. Our simple optimization problem  $\Theta(A_0)$  has convenient properties. First, it is formulated in terms of unconditional expected values.

Second, there are no path dependent constraints. Third, all stochastic terms are unconditionally distributed with *a priori* known distribution. Pereira and Zhang (2004) computed both, a static and a dynamic programming solution for a model similar to ours. They find that the difference of the two procedures is numerically irrelevant. We further argue that the theoretically superior dynamic programming method may lead to worse numerical results in our concrete application, compared to the robust static method. For example, the numerical dynamic programming procedure for a model with continuous state space and continuous control space used by Pereira and Zhang (2004), approximates each state variable and each control variable with a discrete grid. Only the grid nodes are considered in the backward recursion. In between the nodes, the value function is interpolated, and outside the grid the value function is extrapolated. The grid may not be too close due to computational constraints resulting in approximation errors. Another source of errors is the approximation of the expected value using some sort of numerical integration. These two types of approximation errors are likely to be severe in our model, since we build on the collective model of risk theory, resulting in a heavily right skewed aggregated claim amount distribution. We therefore choose the robust static optimization procedure described above.

### 3.2 Optimal Liquidation Strategies

In this section we analyze the liquidation strategy of an insurance company in illiquid markets. Using the model framework introduced in the previous sections, we focus on the influence of temporary and permanent price impact on the optimal liquidation strategy of an insurance company with given initial asset allocation. More precisely, we study different parameter settings for the two price impact functions and the initial asset allocation, and analyze their influence on the optimal liquidation strategy. The optimal

strategy is computed using the inner loop of the optimization procedure described above.

To better understand the impact of illiquidity on the optimal liquidation strategy, we start with a reference case in a perfectly liquid market. In this base case, both the permanent price impact parameter  $\psi = 0$  as well as the temporary price impact parameter  $\phi = 0$  are set to zero. We then optimize the liquidation strategy  $N_t, t = 1, \dots, 12$  for fixed initial asset allocations  $N_0$ . Recall,  $N_t$  denotes the number of units of the risky stock the insurance company is holding at time  $t$ .

- Please insert FIGURE 3 approximately here -

Figure 3 shows optimal liquidation strategies for selected initial asset allocations in the base case without price impact. To improve the readability of the graph, we present the percentage of stock holdings in relation to the overall assets at the beginning of the period under consideration,  $N_t/A_0, t = 0, \dots, 12$ , instead of the absolute number  $N_t$ . Let us have a closer look at these strategies first. With 100% invested in the stock, the insurance company has to liquidate a fraction of the stock holding every period in order to settle the insurance claims. An insurance company with 50% of its assets allocated in the stock, and 50% allocated in cash, can use the cash position first, to pay the claims. This order is optimal for our risk neutral insurance company, because the stock earns a higher expected return than the cash position. The insurance company, hence, does not start to liquidate the stock holding until there is no cash left. An initial asset allocation of 36.65% in the stock leaves in cash the equivalent of the present value of the expected claim payments. This cash position is sufficient for the claim settlement, as the whole model is formulated from an expected value point of view. Concerning the stock position, the insurance company does not have to sell any of them, resulting in

a kind of "buy and hold" strategy. The same argumentation holds for any initial asset allocation with less than 36.65% in the stock and more than the present value of the expected claim payments in cash, respectively.

Now, we introduce price impact, leaving all other parameters unchanged. To separate the effects of the temporary and the permanent price impact, we set one of the two to zero and vary the other one. Let us start with the case of an initial asset allocation whose cash position is greater than or equal to the present value of the expected claim payments. In this case it is optimal for the insurance company to pay its liabilities in cash and to hold its complete stock position. This result does not change regardless of which price impact model is used. So, in this case price impact has no influence on the optimal liquidation strategy. This result is not surprising, since price impact only will make a difference if there are transactions.

- Please insert FIGURES 4 and 5 approximately here -

When the insurance company has 100% of its initial assets invested in the stock, it has to liquidate a fraction of the stock holding every period to pay the claims. Figure 4 shows the influence of temporary price impact on the optimal liquidation strategy. Temporary price impact can be seen as transaction costs for individual trades. The insurance company, thus, has to sell a slightly bigger portions of its stock position each period, to finance the additional expenditure. However, with permanent price impact the present transaction does not only influence the present transaction price, but also changes the value of the remaining stock position, and thus influences all future transaction prices. Focusing on one single trade, the insurance company has to compensate two effects. First, it has to finance the additional expenditure resulting from the price impact of

the present trade, and second, it has to consider that the actual stock holding has less value, due to the price impact of previous trades. These effects accumulate over time, and hence the additional fraction of the stock holding the insurance company is selling in order to finance these effects, is growing from period to period. Figure 5 shows the influence of permanent price impact on the optimal liquidation strategy. Note that the selected levels of price impact in figures 4 and 5 differ significantly. The parameter values were chosen to best visualize the discussed effects.

- Please insert FIGURES 6 and 7 approximately here -

Let us now consider the case for which the insurance company has less than 100%, but more than 36.65% of its initial assets invested in the stock. This means that the cash position is greater than zero but less than the present value of the expected claim payments. The insurance company can therefore pay some of the claim with cash, but has to liquidate fractions of the stock position to pay the remaining claims. Figure 6 shows the influence of temporary price impact on the optimal liquidation strategy. As long as there is still cash left, it is optimal for the insurance company to use the cash first. When the insurer starts liquidating the stock position, it has to sell a slightly bigger portion of it each period, to finance the additional transaction costs. With price impact, there are two opposite effects. On the one hand, the stock earns a higher expected rate of return than the cash position, making it favorable for the insurance company to hold the stock position as long as possible. On the other hand, the price impact depends on the trading volume, making it favorable for the insurer to split a big transaction in multiple small ones, and hence start earlier with the liquidation of its stock position. With temporary price impact the first effect dominates, however, with permanent price

impact the second effect gets more and more important. Figure 7 shows the influence of permanent price impact on the optimal liquidation strategy for selected levels of price impact. It is now optimal for the insurance company to start the liquidation of its stock position before it runs out of cash. With growing permanent price impact the trading volume is split more and more evenly over the considered time periods. We also find that the permanent price impact has a much stronger impact on the optimal trading strategy as the temporary price impact. Note that the permanent price impact parameter values used in figure 7 are significantly smaller than the ones for figure 6.

### 3.3 Optimal Asset Allocation

The previous analysis focused on optimal liquidation strategies for given initial asset allocations which corresponds to the inner loop of our optimization procedure. We now use the complete two loop optimization to examine which initial asset allocation is optimal for the insurance company.

Once again, we start our discussion with the reference case of a perfectly liquid market. In a world without price impact, a risk neutral investor should invest 100% of its wealth in the asset with the highest expected return. Thus it is optimal for the insurance company in our model to allocate 100% of its initial assets in the risky stock.

- Please insert FIGURES 8 and 9 approximately here -

Let us now study the influence of different levels of temporary price impact on the optimal initial asset allocation. As we see in figure 8, for growing temporary price impact it is optimal for the insurance company to reduce the fraction of the stock in the asset

allocation. But it is not optimal to make the stock holding arbitrarily small. There is a limit, and the stock position converges to that limit for growing price impact. Recall the discussion of optimal liquidation strategies in the last section. When the cash position in the asset allocation is greater than or equal to the present value of the expected claim payments, it is optimal for the insurance company to pay its liabilities in cash and to hold its complete stock position. This optimal strategy is not sensitive to the level of price impact. Due to the fact that the stock earns a higher expected return than cash, the insurance company should reduce its cash position exactly to the present value of the expected claim payments, and invest the complete remaining amount in the stock. Summing up, for growing temporary price impact, the optimal stock position converges from the top to one minus the present value of expected claim payments. The same argumentation holds for the influence of permanent price impact on the optimal initial asset allocation. However, there is one big difference, the influence of permanent price impact on the optimal asset allocation is much stronger. This results in a faster speed of convergence, as visualized in figure 9. Note that the permanent price impact parameter values used in figure 9 are significantly smaller than the ones for figure 8.

Let us summarize: the optimal asset allocation converges to a limit for increasing price impact. In the limit it is optimal to hold the expected value of the insurance claims in cash and invest the remaining money in the stock market. The corresponding optimal liquidation strategy for the insurance company is a "buy and hold" strategy. This means it is optimal for the insurance company to hold the complete stock position and pay all insurance claims with cash. This strategy works out, since our analysis is solely from the aspect of expected value. Thus, the insurance company can pay all *expected* claims with cash and does not have to liquidate its stocks. This behavior can be interpreted as the realization of a cash-flow matching strategy. The optimal solution set we derive here, is therefore in line with the literature, since for example Elton and Gruber (1992) proved

the cash-flow matching strategy to be ideal for hedging the liabilities of an institutional investor.

### 3.4 Stability of Results

An example for the type of insurance modelled in this initial design is a property and casualty insurance company with an industrial fire insurance business line. Industrial fire insurance is a low frequency and high severity business meaning that claims are high, but relatively infrequent. To show that the results do not depend on the type of insurance business, we repeated the analysis for a standard high frequency, low severity business line like for example automobile insurance. Assuming that the claim amounts follow a gamma distribution our second line of business is much less "dangerous". Following the example of Kaufmann, Gadmer, and Klett (2001) (see p. 241), we suppose that the claim size measured in US-\$ follows a gamma distribution  $U_i \sim \Gamma(a, b)$  with shape parameters  $a = 9.091$  and scale parameter  $b = 242$ . In our model the inter-occurrence times between claims measured in months are exponentially distributed  $T_i \sim Exp(\lambda)$ . We now use the parameter value  $\lambda = 0.00208$ . This implies that in the long run there are on average 481 claims per month or 5,769 per year. This results in smaller and more frequent claims. We endow the insurance company with enough financial assets to ensure that the company can pay their liabilities more than 99.5% of the time. All other parameters take values as specified before. To cut a long story short, the qualitative properties of the optimal initial asset allocation and liquidation strategy are the same. Only the magnitude with which price impact influences the asset allocation and liquidation strategy is smaller for a given parameter setting. We attribute this to the fact that the trading volume is smaller in this case.

To check, whether an increased trading volume *ceteris paribus* changes our results,

we doubled the financial assets as well as the insurance claims for the industrial fire insurance business line, and rerun the analysis. In this case, the qualitative properties of the optimal initial asset allocation and liquidation strategy are unaffected, as well. However, let us have a close look on the magnitude of the effect. Since the temporary price impact can be seen as transaction cost, it simply reduces the expected return of the stock. Increasing the trading volume therefore results in higher absolute costs, but the relative costs stay unchanged. Thus increasing the transaction volume does not effect the speed of convergence of the optimal stock holding to its limit. The speed is only determined by the ratio between the expected stock return and the temporary price impact parameter. This relationship is completely different for the permanent price impact. Increasing the trading volume, here, results in a higher impact on the stock price, and this effect accumulates over time. Hence an increase in trading volume results in a faster convergence of the optimal stock holding to its limit. This means that for bigger insurance companies with bigger trading volumes, a cash-flow matching strategy is preferable, for even minimal permanent price impact.

## 4 Implications for Asset Management

In this section, we discuss the implications of our results for the asset management decisions of a property and casualty insurance company. Our analysis uses a wide range of values for the temporary and permanent price impact parameters, and studies their impact on the optimal initial asset allocation and liquidation strategy. Let us now focus on the empirical magnitude of these parameters.

Hausman, Lo, and MacKinlay (1992), and Kempf and Korn (1999) in their empirical studies find an overall price impact of about  $10^{-5}$  for large transactions in liquid stocks. Brennan and Subrahmanyam (1996) analyze all NYSE-listed securities included on the

CRSP data tape and estimate for each of them the overall price impact parameter, Kyle's  $\lambda$ , for the years 1984 and 1988. They find an overall price impact parameter of  $10^{-5}$  for transactions in liquid stocks and price impact parameters up to  $10^{-3}$  for illiquid stocks. Holthausen, Leftwich, and Mayers (1990) examine the 50 largest buy and sell trades in 1983 for 109 randomly selected NYSE-listed firms. They find that for selling transactions, on average, 85% of the price impact is permanent. Chan and Lakonishok (1995) analyze all trades executed by 37 large investment management firms between July 1986 and December 1988. They focus on the price impact of sequences of trades, so called block trades. They find that for sales approximately 50% of the price impact is permanent.

Permanent price impact adversely influences the future stock price. But a negative future stock price development only influences an insurance company's portfolio value, when the company still holds a position of this stock. If the insurance company liquidates a stock position completely, permanent price impact only results in temporary transactions costs. Therefore, it is advantageous for the insurance company to liquidate a stock position in one block sale. This means that there is an incentive for the insurance company to require each individual asset position in its portfolio to be small enough for a single block sale. Summing up, there is a diversification benefit in illiquid markets apart from the one due to market risk considerations introduced by Markowitz (1952). However, this argumentation is only correct, if we assume that there are no commonalities of permanent price impact across different stocks. With such commonalities a sale of the insurance company's IBM shares may affect the value of the insurance company's GE shares. There are some empirical studies, examining commonalities in different measures of liquidity. This literature includes the work of Chordia, Roll, and Subrahmanyam (2000), and the subsequent work of Halka and Huberman (2001), Hasbrouck and Seppi (2001), Brockman and Chung (2002), and Kempf and Mayston (2005). But none of

these studies examines the commonalities of permanent price impact. Since there is empirical evidence for return correlations of different stocks as well as for the overall price impact, we argue that there is commonality of permanent price impact. For example Kempf and Mayston (2005) find return correlations of about 30% between major stocks of the German DAX index. For our further discussion we assume the commonality of permanent price impact to be 15%. Figure 10 plots the insurance company's expected surplus for different breakdowns of the overall price impact into permanent and temporary portions. As we see, a reduction in permanent price impact results in an expected surplus greater than or equal to the initial one. This result holds for all possible asset allocations. Thus, even when there is commonality in permanent price impact, and the insurance company cannot eliminate the influence of its sales transactions on its remaining portfolio positions completely, it is still worthwhile to reduce this influence as far as possible. The insurance company should diversify its portfolio in positions small enough to be sold in one block transaction.

- Please insert FIGURE 10 approximately here -

Whether it is optimal for an insurance company to adopt a cash-flow matching strategy depends on the existence of commonality in permanent price impact. If our model insurance company can eliminate the impact of its sales transactions on the value of its remaining portfolio, completely, it will be optimal to invest 100% of its assets in stocks. But, as Figure 10 shows, an effective permanent price impact of 15% makes it optimal for the model insurer to apply a cash-flow matching strategy. One can argue that the assumed commonality of 15% is much too high. But for increasing trading volumes, the permanent price impact parameter  $\psi$  can be smaller and it is still optimal for the model

insurance company to apply cash-flow matching. Let us keep in mind that the model insurer is a very small one with only one line of business (see section 3). Therefore we argue that if there is commonality in permanent price impact, it is optimal for standard multiple line insurance companies to apply cash-flow matching.

## 5 Conclusion

In this paper, we study the impact of illiquidity on optimal asset allocation decisions and liquidation strategies of property and casualty insurance companies. We propose a cash-flow based liquidation model of an insurance company, assuming that the insurance company is risk neutral and optimizes its expected terminal surplus. Deviating from perfectly liquid market condition, the insurance company in our model has to make a price concession when selling securities.

We distinguish between two types of price impact: temporary and permanent. Permanent price impact adversely influences the future stock price. Temporary price impact, however, can be seen as a transaction cost for a single trade. Whether a negative future stock price development influences an insurance company's portfolio value depends on whether the company still holds a position of this stock. If the insurance company liquidates a stock position completely, permanent price impact only results in temporary transactions costs. This assumes that there is no commonality in price impact. However in the presence of commonality, liquidations of complete stock positions still result in a significantly reduced permanent price impact on the portfolio value.

We numerically compute the optimal liquidation strategy and asset allocation of an insurance company. We show that the negative influence on the optimal liquidation strategy and asset allocation is much stronger for permanent price impact than for

temporary price impact. It is therefore advantageous for an insurance company to liquidate stock positions in one block sale. For this to be possible each individual asset position in the insurance company's portfolio has to be small enough for a single block sale, resulting in a well diversified portfolio. Since our model is formulated in terms of expected value, it abstracts from market risk considerations, and thus shows that there is a diversification benefit which can be attributed only to illiquidity. This result shows that diversity is not only advantageous for mitigating the market uncertainty of an individual stock. Diversity is a rational reaction for a large investor wishing to minimize the negative impact of present trades on future portfolio value.

We further show that the optimal asset allocation converges to a limit for increasing price impact. In the limit, it is optimal to hold the expected value of the insurance claims in cash and invest the remaining money in the stock market. The insurance company can pay all expected claims with cash and does not have to liquidate stocks. This behavior can be interpreted as the realization of a cash-flow matching strategy.

Whether it is optimal for an insurance company with a well diversified portfolio to apply a cash-flow matching strategy, depends on the existence of commonality in permanent price impact. Without such commonality, the insurance company can eliminate the impact of its sales transactions on the value of its portfolio completely. It is then optimal to invest 100% of its assets in stocks. But with commonality in permanent price impact, even for well diversified insurance companies, there is a permanent price impact on the company's portfolio value. In this case, it is optimal for the insurer to apply a cash-flow matching strategy. Whether there is commonality in permanent price impact and, given that commonality exists, of what magnitude is still an open research question.

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## Notes

<sup>1</sup>For additional empirical evidence about price impact in security markets, see Kraus and Stoll (1972), Holthausen, Leftwich, and Mayers (1987), Hasbrouck (1991) Chan and Lakonishok (1993), Chan and Lakonishok (1995), Knez and Ready (1996), Kempf and Korn (1999), Engle and Lange (2001) and Spierdijk, Nijman, and Van Soest (2004).

<sup>2</sup>Vayanos (2001) uses the terms "informational motives" and "allocational motives" to distinguish between different kinds of trading motives.

<sup>3</sup>Examples of models examining dynamic strategies of traders with informational motives include Glosten and Milgrom (1985), Foster and Vishwanathan (1996) and Back, Cao, and Willard (2000). See also the book by O'Hara (1995).

<sup>4</sup>The collective model of risk theory is the standard actuarial risk model. It is based on the fundamental work of Lundberg (1903). For an introduction to the collective risk model see Gerber (1979), Klugman, Panjer, and Willmot (1998), or Rolski, Schmidli, Schmidt, and Teugels (1999).

<sup>5</sup>The inaccuracy we introduce here is quite small, especially for short tailed lines of business.

<sup>6</sup>Specifically Huberman and Stanzl (2004) showed that when the price impact of trading activity is time stationary, only linear price impact functions rule out quasi-arbitrage. However, when the temporary and permanent price effect of trades are independent, only the permanent price impact must be linear.

<sup>7</sup>This assumption does not restrict the applicability of this model in a real word situation because any investor with access to the Electronic Communication Network

observes the whole order book.

<sup>8</sup>We simulated the aggregated annual claim amount 100,000 times. This number is less than 150,000,000 US-\$ in 99,587 of the 100,000 scenarios.

<sup>9</sup>The solution  $N_t, t = 0, 1, \dots, 12$  of the overall optimization is a vector in the  $\mathbb{R}^{13}$ . By fixing the initial asset allocation, we reduce the dimension of the optimization from 13 to 12. Since we do not allow reinvestments ( $N_{t+1} \leq N_t, t = 0, 1, \dots, T - 1$ ) and short selling ( $N_t \geq 0, t = 0, 1, \dots, T$ ), we know that the optimal solution lies in a small subset of the  $\mathbb{R}^{12}$ . Restricting the optimization to a smaller subset always reduces cpu-time and *ceteris paribus* improves accuracy. By splitting the overall optimization procedure in such a two loop procedure, we can use our additional information about the properties of the solution, look for an optimum in a restricted subspace, and hence improve the result.

<sup>10</sup>A sample implementation of the downhill simplex method can be found in Press, Teukolsky, Vetterling, and Flannery (2002).

<sup>11</sup>We use an algorithm similar to the Fortran program of the Golden Section search with parabolic interpolation in Forsythe, Malcolm, and Moler (1976).

Figure 1: Temporary, Permanent and Total Price Impact

This figure shows the temporary and permanent price impact elements.  
 (See Holthausen, Leftwich, and Mayers (1987), Fig. 1)

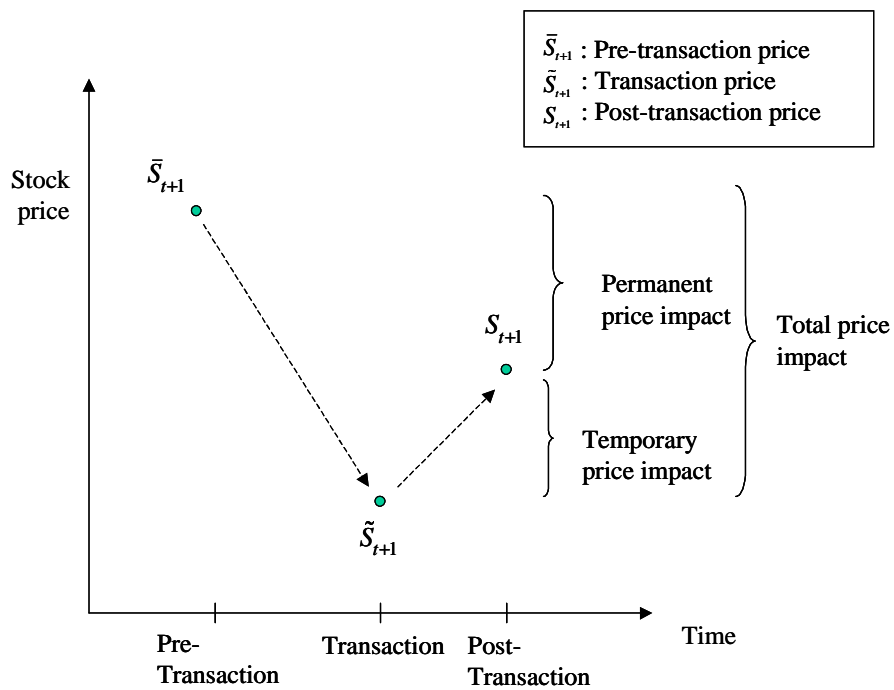
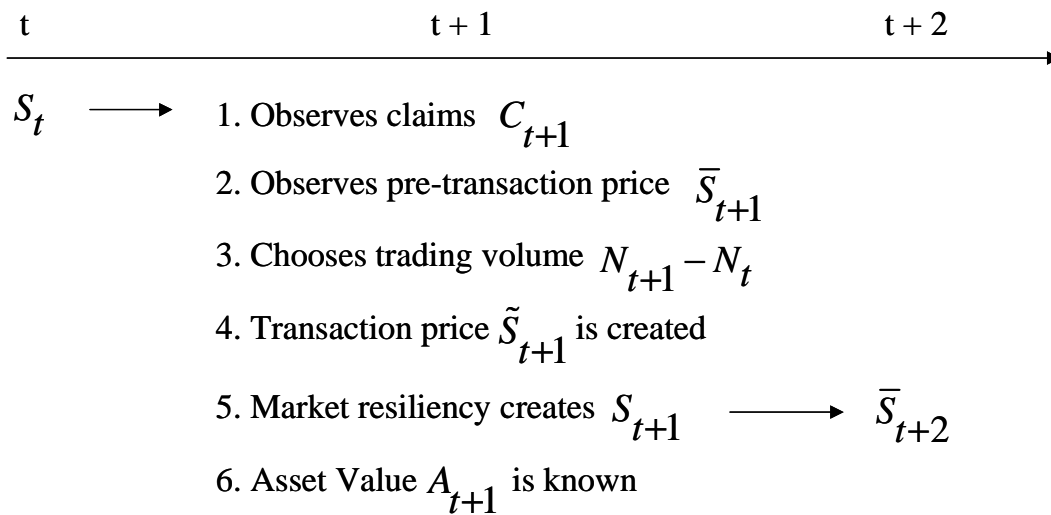


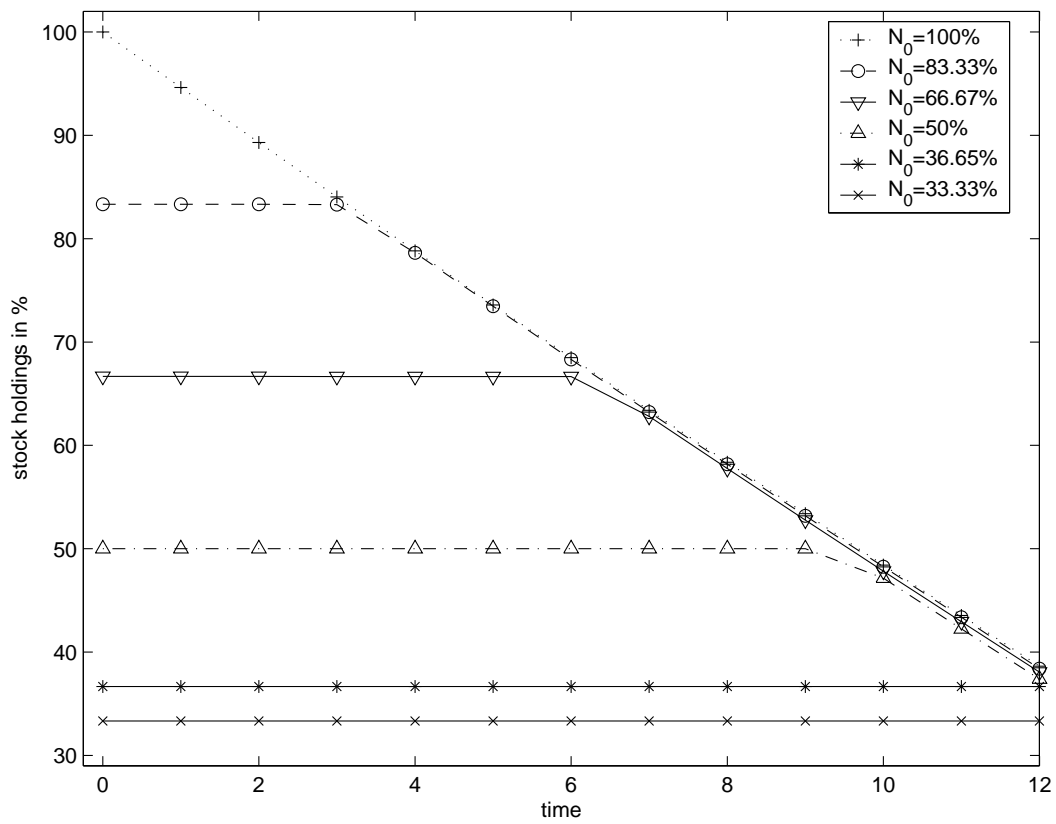
Figure 2: Timing of the Model

This figure shows the time scheme of the model.



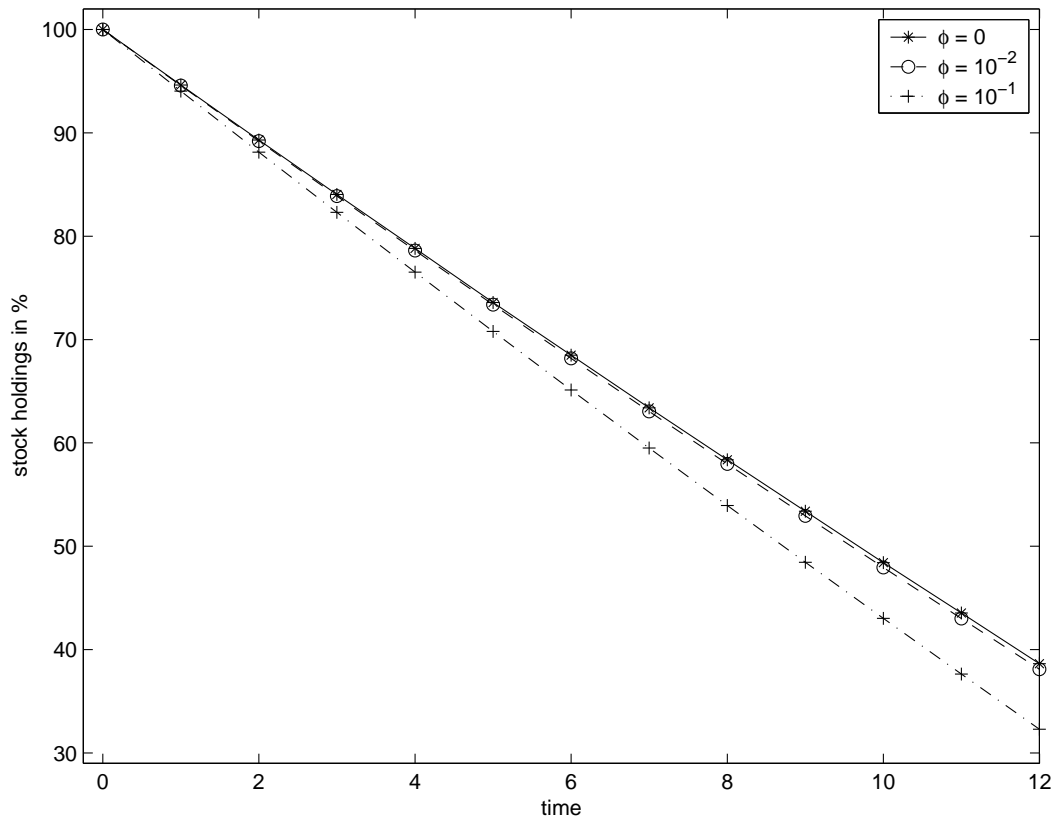
**Figure 3: Optimal Liquidation Strategies - No Price Impact**

This figures shows optimal liquidation strategies for different initial asset allocations  $N_0$  in the base case without price impact.



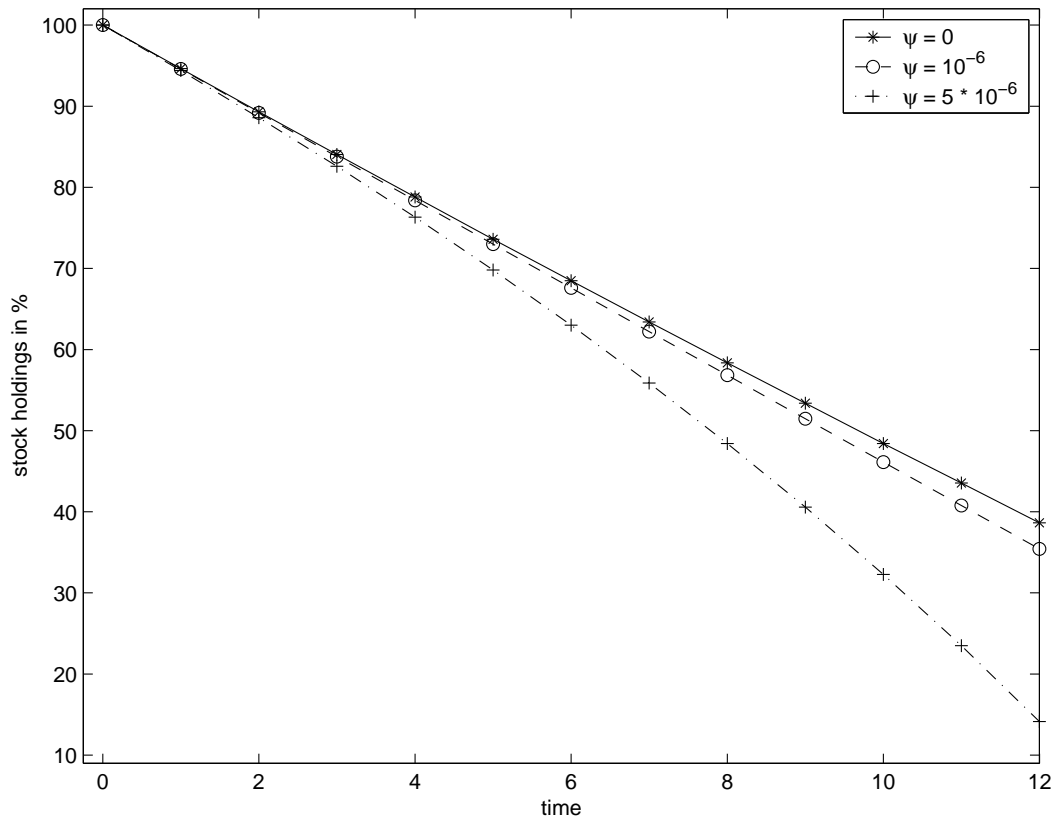
**Figure 4: Optimal Liquidation Strategy with Temporary Price Impact - 100% in Stock**

This figure shows the influence of temporary price impact on the optimal liquidation strategy. The initial asset allocation is 100% in stock.  $\phi$  is the temporary price impact parameter.



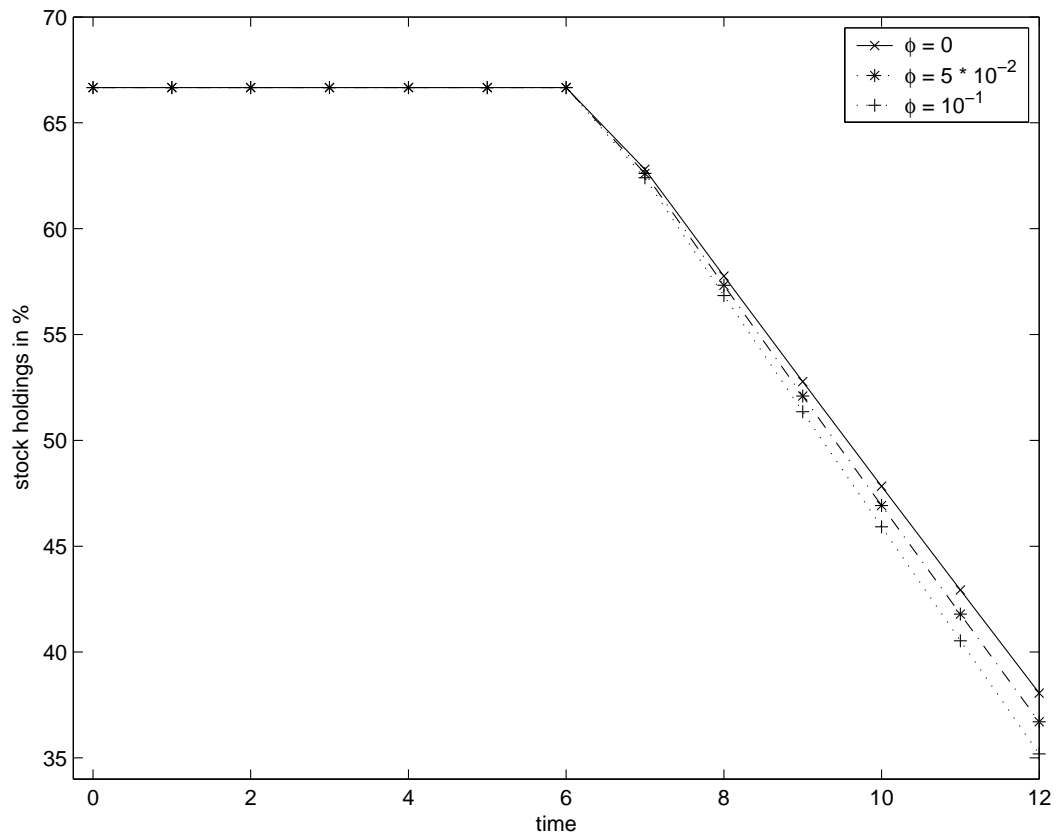
**Figure 5: Optimal Liquidation Strategy with Permanent Price Impact - 100% in Stock**

This figure shows the influence of permanent price impact on the optimal liquidation strategy. The initial asset allocation is 100% in stock.  $\psi$  is the permanent price impact parameter.



**Figure 6: Optimal Liquidation Strategy with Temporary Price Impact**

This figure shows the influence of temporary price impact on the optimal liquidation strategy. The initial asset allocation is 66.67% in stock.  $\phi$  is the temporary price impact parameter.



**Figure 7: Optimal Liquidation Strategy with Permanent Price Impact**

This figure shows the influence of permanent price impact on the optimal liquidation strategy. The initial asset allocation is 66.67% in stock.  $\psi$  is the permanent price impact parameter.

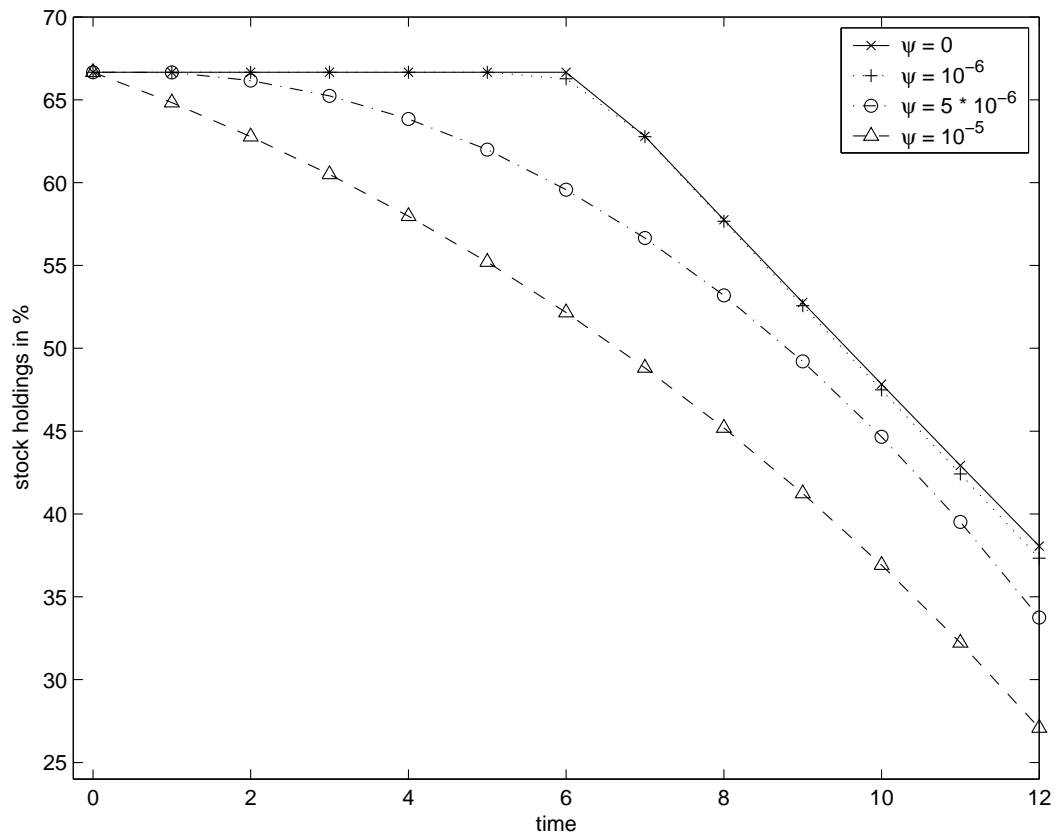


Figure 8: Optimal Asset Allocation with Temporary Price Impact

This figure shows the influence of temporary price impact on the expected surplus for all possible initial asset allocations.  $\phi$  is the temporary price impact parameter. Without price impact ( $\phi = 0$ ) the expected surplus is maximal for allocating 100% of the assets in stock. For growing temporary price impact the optimal asset allocation converges to 36.65% in stock, which corresponds to one minus the expected value of claim payments.

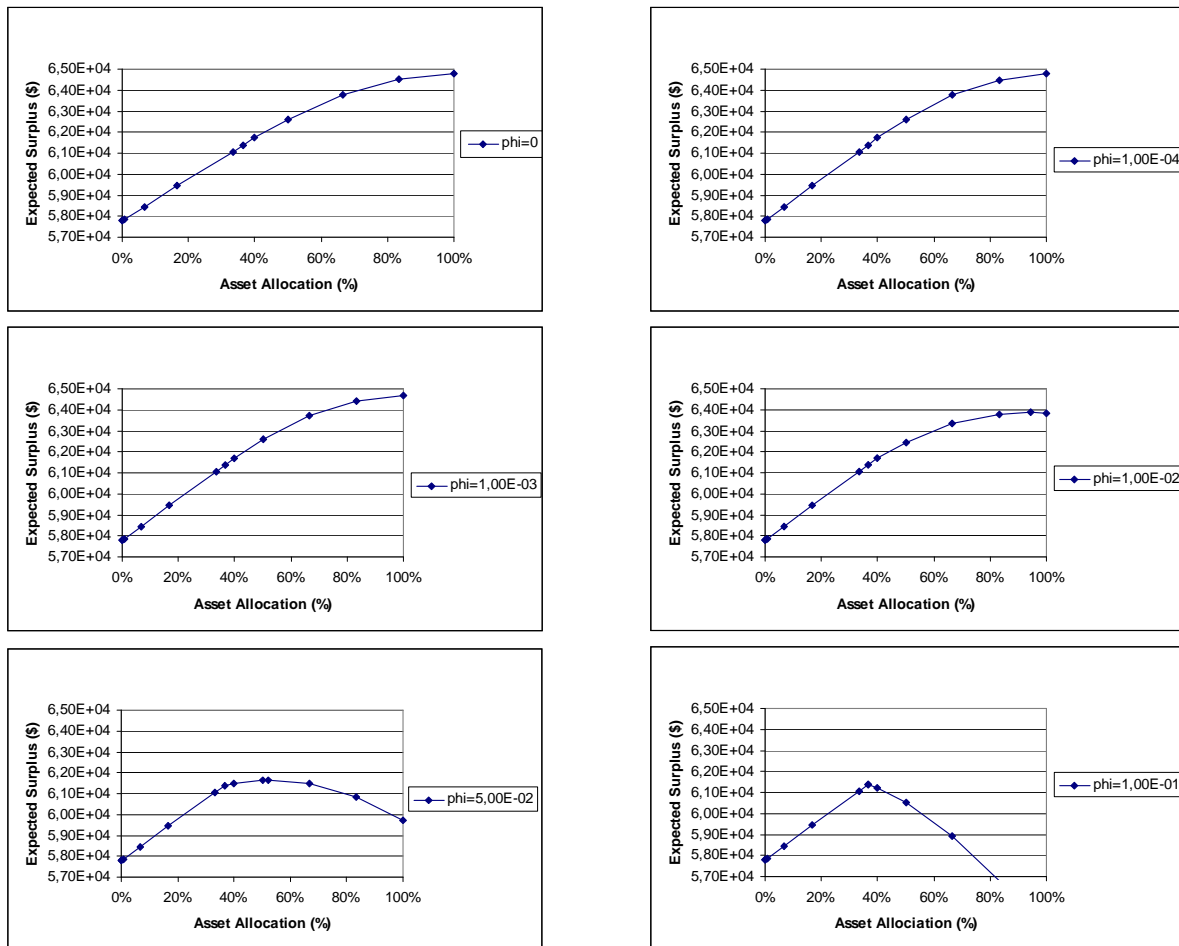
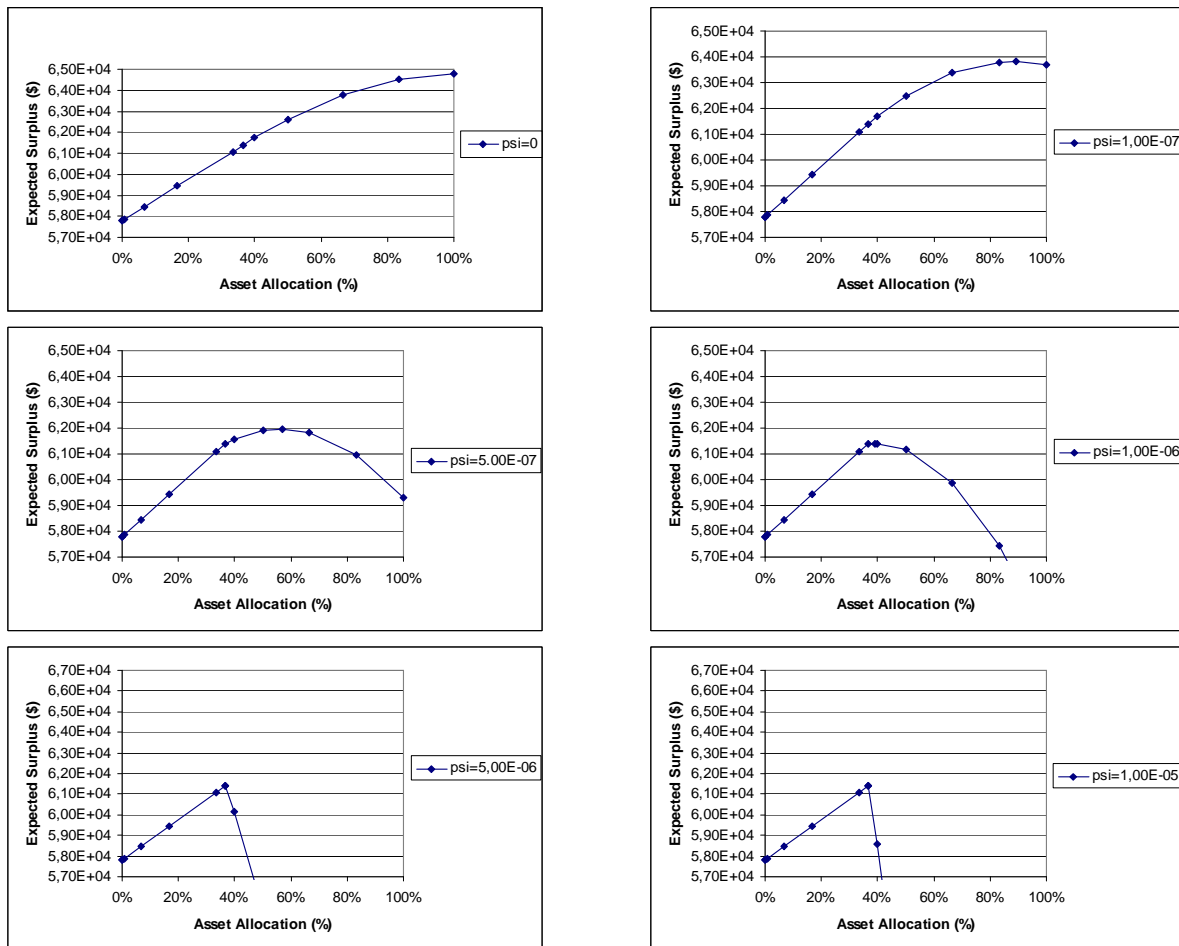


Figure 9: Optimal Asset Allocation with Permanent Price Impact

This figure shows the influence of permanent price impact on the expected surplus for all possible initial asset allocations.  $\psi$  is the temporary price impact parameter. Without price impact ( $\psi = 0$ ) the expected surplus is maximal for allocating 100% of the assets in stock. For growing temporary price impact the optimal asset allocation converges to 36.65% in stock, which corresponds to one minus the expected value of claim payments.



**Figure 10: Variation in Temporary Price Impact**

This figure shows the expected surplus for all possible initial asset allocations under various ratios of permanent and temporary price impact. The overall price impact is set to  $10^{-5}$ . If 100% of the price impact is temporary the expected surplus is maximal for allocating 100% of the assets in stock. At permanent price impact percentages over 15% it is ideal to hold 36.65% in stock, which corresponds to one minus the expected value of claim payments.

