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The Impact of Surplus Distribution on the Risk Exposure of With Profit Life Insurance Policies Including Interest Rate Guarantees

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The Impact of Surplus Distribution on the Risk Exposure of With Profit Life Insurance Policies Including Guarantees

Alexander Kling, Andreas Richter, Jochen Russ

Abstract

This paper analyzes the impact of different surplus distribution mechanisms on the risk exposure of a life insurance company selling with profit life insurance policies with a cliquet-style interest rate guarantee. Three different types of distribution mechanism are considered: A mechanism, where the guaranteed interest rate also applies to surplus that has been credited in the past, a slightly less restrictive mechanism in which the a guaranteed rate of interest of 0% applies to past surplus, and a third mechanism that allows for the company to use former surplus in order to smoothen underperformance in "bad" years. Our study demonstrates that regulators should be very careful in deciding which design of a distribution mechanism is to be enforced. Results strongly suggest that a distribution mechanism of the third type is in many ways superior to the other two mechanisms. In particular, throughout the analysis, our representative company 3 faces ceteris paribus significantly lower shortfall risk than the other two companies. Requiring "strong" guarantees puts companies at a significant competitive disadvantage relative to insurers which are subject to regulation that only requires the company 3 type surplus distribution mechanism. This is particularly true, if annual minimum participation rates are required for long term contracts.

1. Introduction

Many with profit life insurance policies contain an interest rate guarantee. Often, this guarantee is given on a point-to-point basis, i.e. the guarantee is only relevant at maturity of the contract. Other products (which are predominant, e.g., in the German market), however, offer a so-called cliquet-style (or year-by-year) guarantee. This means that the policy holders have an account to which every year a certain rate of return has to be credited. Typically, life insurance companies try to provide the guaranteed rate of interest plus some surplus on the policy holders' accounts.

There are different mechanisms defining how the annual surplus can be distributed to the insured. These mechanisms vary from country to country and sometimes from insurance company to insurance company. They can be divided in three different categories and combinations thereof:

a) Surplus may be credited to the policy reserves. In this case, it is guaranteed that this surplus will earn the guaranteed rate of interest in future years.

b) Surplus may be credited to some surplus account that is owned by the policy holder and may therefore not be reduced anymore. Thus, there is a guaranteed interest rate of 0% on money that is in this surplus account.

c) Surplus may be credited to some terminal bonus account. Money that has been credited to this account will only be distributed to the insured at maturity of their contracts but not if they cancel the contract. Furthermore, money may be taken from this account in order to pay interest rate guarantees if in some year the return on assets is not sufficient to pay for these guarantees.

It is obvious that c.p., insurance companies using different surplus distribution mechanisms may have a significantly different risk profile. In the past, this may have been of minor importance since there was a comfortable margin between market interest rates and the guaranteed rates that were typically offered within life insurance policies. Recently, however, the difference between market interest rates and guaranteed rates (in particular guaranteed rates of contracts that have been sold years ago) has been significantly reduced. This development illustrates that analyzing and managing an insurance company's financial risks should not only be restricted to management of the assets but also be concerned with reducing risks that result from the product design.

A number of papers have recently addressed interest rate guarantees, in particular Briys and de Varenne (1997), Grosen and Jorgensen (2000), Jensen et al. (2001), Miltersen and Persson (2001), Hansen and Miltersen (2002), Grosen and Jorgensen (2002), Bacinello (2003), Tanskanen and Lukkarinen (2003), and Kling, Richter and Russ (2004).

Briys and de Varenne (1997) compute closed-form solutions for market values of liabilities and equities in a point-to-point guarantee framework. In their model the policy owner receives a guaranteed interest and is also credited a bonus, determined as a certain fraction of net financial gains (when positive). The paper also looks at the impact of interest rate guarantees on the company's risk exposure by analyzing interest rate elasticity and duration of insurance liabilities.

Contrasting the just-mentioned approach, Grosen and Jorgensen (2000) consider cliquet-style guarantees and introduce a model that takes into account an insurer's use of the average interest principle. In addition to a policy reserve (the customer's account) they introduce a "bonus reserve", a buffer that can be used to smoothen future bonus distributions. They analyze a mechanism that credits bonus to the customer's reserve based upon the current ratio of bonus reserve over policy reserve. A bonus is paid only if this ratio exceeds a given threshold. Thus, the actual distribution of surplus indirectly reflects current investment results but primarily focuses on the company's ability to level out insufficient results in the future. The authors decompose the contract into a risk free bond, a bonus and a surrender option. They compute contract values by means of Monte Carlo simulation, and also calculate contract default probabilities for different parameter combinations.[1] However, they calculate default probabilities under the risk neutral probability measure Q. Therefore, the numerical results are of only limited explanatory value.

Miltersen and Persson (2001) also use a cliquet-style framework and allow for a portion of excess interest to be credited not directly to the customer's account but to a bonus account. In their model, the interest that exceeds the guaranteed rate is – if positive – divided into three portions that are credited to the insured's account, the insurer's account, and to a bonus account.

[1] Jensen et al. (2001) extend the findings of Grosen and Jorgensen (2000). As one extension, among others, they introduce mortality risk. Another paper that incorporates mortality risk as well as the surrender option is Bacinello (2003).

In case of investment returns below the guaranteed rate, funds are moved from the bonus account into the policy owner's account. Thus, the bonus account is available for smoothing returns over time. Unlike in the Grosen and Jorgensen (2000) model, however, the buffer consists of funds that have already been designated to the particular customer: Any positive balance on the bonus account is credited to the policy owner when the contract expires. This is used to model so-called "terminal bonuses". In this setting, Miltersen and Persson (2001) derive numerical results on the influence of various parameters on the contract value.[2]

Grosen and Jorgensen (2002) discuss a model based upon the framework used by Briys and de Varenne (1997). They incorporate a regulatory constraint for the insurer's assets according to which the company is closed down and liquidated if the market value of assets drops below a threshold at any point in time during the life of the policy. Their results suggest that the introduction of the regulatory constraint significantly reduces the value of the shareholders' default put option and thereby an insurer's incentive to change its assets' risk characteristics to the policyholders' disadvantage.

While the primary focus in the literature is on the fair (i.e. risk-neutral) valuation of the life insurance contract, Kling, Richter and Russ (2004)[3] concentrate on the risk a contract imposes on the insurer, measured by means of shortfall probabilities under the so-called "real-world probability measure P". Assuming cliquet-style guarantees, they study the impact interest rate guarantees have on the exposure of the insurance company and how default risks depend on characteristics of the contract, on the insurer's reserve situation and asset allocation, on management decisions, as well as on regulatory parameters.

The present paper analyzes the impact of different surplus distribution mechanisms on the risk exposure of a life insurance company selling with profit life insurance policies with an interest rate guarantee. We employ the model framework introduced in Kling, Richter and Russ 2004, but extend the model such that the different surplus mechanisms described above and any combinations can be compared. The model also allows for the comparison of the two major types

2 Contrasting the mechanism discussed in Miltersen and Persson (2001), life insurance contracts often employ a distribution policy that does not accumulate undistributed surplus on an individual basis, but for a greater pool of customers. A model that allows for this technique can be found in Hansen and Miltersen (2002).

3 Kling, Alexander, Richter, Andreas and Russ, Jochen (2004): The Interaction of Guarantees, Surplus Distribution, and Asset Allocation in With Profit Life Insurance Policies. Working Paper.

of interest rate guarantees: cliquet-style guarantees and point-to-point guarantees, as described above. The focus of our numerical analysis, however, is on the different surplus distribution mechanisms.

2. The model framework

2.1 The insurer's financial situation

In our model, we use a simplified illustration of the insurer's balance sheet. We expand the model introduced in (Kling/Richter/Russ 2004) so that different surplus distribution mechanisms can be included:

Assets	Liabilities
A_t	E_t $\left.\begin{array}{l}L_t\\ S_t\\ B_t\end{array}\right\}$ policy holders' accounts R_t reserves
A_t	A_t

Figure 1: Model of the insurer's financial situation

Here, A_t, denotes the market value of the insurer's assets at time t. The liability side comprises five entries:

- E_t is the time t book value of the company's equity. It represents a buffer for the insurance company to meet guarantees.
- L_t is the time t book value of the policy reserves. The insurer guarantees the policy holder an annual interest rate g on this account. Note that any surplus that is credited to this account will also have to earn at least the guaranteed rate in the future.
- S_t is the time t book value of the policy holders' individual surplus accounts. Consistent with German legislation, we assume that the guaranteed rate need not be credited on this account but once surplus is distributed to this account it may not be

reduced at any time in the future, i.e., the guaranteed rate of interest on this account is 0% p.a.

- B_t is the time t book value of the bonus account for terminal bonuses. It is owned by the policy holders' but not on an individual basis. Parts of this account are paid out to maturing contracts as a terminal bonus. It may however also be used to provide guarantees for other accounts in the future.
- R_t is the reserve account which is given by $R_t = A_t - L_t - S_t - B_t$. It consists mainly of asset valuation reserves.

Whenever dividends D_t are paid out to equity holders, A_t is reduced by the corresponding amount. To simplify notation, we assume that such payments occur annually, at times $t = 1,2,\ldots,T$, where T denotes some finite time horizon.

2.2 The asset model

Similar to the approach in Kling, Richter and Russ 2004, we use a very simple model for the assets: We assume a complete, frictionless and continuous market. Between dividend payments, we let A_t follow a geometric Brownian motion

$$\frac{dA_t}{A_t} = \mu(t)dt + \sigma(t)dW_t, \qquad (1)$$

where W_t denotes a Wiener process on some probability space (Ω,Σ,P) with a filtration F, to which W is adapted. Both, μ and σ are deterministic but can be time dependent. The numerical analysis in section 3 assumes A_t to consist of stocks and bonds with s denoting the stock portion of the (continuously rebalancing) portfolio. We let μ = 8% and σ = 20% for the stock portion of the portfolio and μ = 5% and σ = 3.5% for the bond portion of the portfolio. Furthermore, we assume stock and bond returns to be slightly negatively correlated with a correlation coefficient of ρ = -0.1.[4] Thus, drift and volatility of A_t can be calculated for any given asset allocation specified by the parameter s.

[4] As in Kling, Richter and Russ 2004, we used data of a German stock index (DAX) and a German bond index (REXP) of the years 1988 to 2003 to get estimates for drift, volatility and correlation of stocks and bonds. Since

Including dividend payments D_t, we get for $t = 1,2,\ldots,T$

$$A_t^- = A_{t-1}^+ \cdot e^{\int_{t-1}^{t} \mu(s) - \frac{\sigma^2(s)}{2} ds + \int_{t-1}^{t} \sigma(s) dW_s} \qquad \text{and} \qquad A_t^+ = A_t^- - D_t .$$

2.3 Insurance Benefits and Guarantees on the Insurance Liabilities

For the sake of simplicity, our model considers an insurance company in a "steady state": We assume that contracts corresponding to some constant fraction ξ of the insurer's liabilities terminate each year due to maturity, surrender or death. The company pays out the corresponding values of the policy holders' accounts, i.e. $\xi \cdot \left(L_{t-1} + S_{t-1} + B_{t-1}\right)$.[5] We assume that the sum of premiums P_{t-1} collected at the beginning of year t (stemming from new business as well as regular (annual) premium "old" contracts) also equals $P_{t-1} = \xi \cdot \left(L_{t-1} + S_{t-1} + B_{t-1}\right)$.[6]

Since the premiums collected are added to the policy reserves, the value of the policy holders' accounts before guarantee provision and surplus distribution are given by

$$L_t^- = (1-\xi)L_{t-1}^+ + P_{t-1}, \; S_t^- = (1-\xi)S_{t-1}^+ \text{ and } B_t^- = (1-\xi)B_{t-1}^+ .$$

The values L_t^+, S_t^+ and B_t^+ then depend on amount and type of surplus distribution.

At the end of each policy year, the guaranteed rate of interest g is provided to the policy reserves and annual surplus is distributed. By definition of the different accounts (see section 2.1), we get the following lower bounds: $L_t^+ \geq L_t^- \cdot \left(1+g\right)$, $S_t^+ \geq S_t^-$ and $B_t^+ \geq 0$.

The amount of distribution to the different policy holders' accounts and to equity holders each year depends on the earnings on book value as well as decisions made by the company's management. Following e.g. German legislation, we assume that there is a "*minimum*

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historical bond returns seem to be too high compared to current low interest rates, we reduced the drift for the bond portion to 5%.

5 We assume that there are neither gains nor losses due to mortality and thus ignore death benefits that might exceed the value of the policyholder's account. This means that the cost of insurance, i.e. the part of the premium that is charged for the death benefit, is calculated with best estimate mortality rates and therefore exactly covers any death benefits that exceed the policy holders' accounts.

6 Since we ignore death benefits that exceed the policy holders' accounts, of course P_t does not include the cost of insurance.

participation rate" requiring that at least a certain portion δ of the earnings on book values has to be credited to the policy holders' accounts.

Earnings on book value are subject to accounting rules giving insurance companies certain freedom to create and dissolve asset valuation reserves. Following the approach introduced in Kling, Richter and Russ 2004, we assume that at least a portion y of the increase in market value has to be identified as earnings in book values in the balance sheet.[7] The parameter y therefore represents the degree of "*restriction in asset valuation*" immanent in the relevant accounting rules. Furthermore, the insurer can reduce reserves in order to increase the book value of assets without any restrictions by selling assets with market value exceeding the book value.

2.4 Surplus distribution and dividend payments

This section deals with the amount of surplus that is credited to the policy holders in any given year, whilst the next section introduces different surplus distribution mechanisms. Surplus that is distributed to the policy holders' accounts and dividends that are paid to the shareholders are determined by the insurance company's management every year. Our general model allows for any management decision rule at time t that is F_t-measurable. In the numerical analysis, however, we will focus on one decision rule that seems to prevail in Germany: In the past, insurance companies used to keep surplus distribution to policy holders and dividends to shareholders rather constant over years, building and dissolving reserves in order to smoothen returns. Only when the reserves reached a rather low level, they started reducing surplus. Therefore, we apply a decision rule that considers this: As long as reserves are in a "comfortable" range, some constant target rate is credited to policy holders' accounts. If crediting this target rate would lead to an "uncomfortably low" reserve level, surplus is reduced. If reducing surplus is not sufficient, first reserves are further dissolved and then the bonus account is reduced. On the other hand, if crediting the target rate would yield to a very high level of reserves, surplus is increased above the target rate. The technical details of this simple idea are explained in the remainder of this section:

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[7] This means that the sum of the increase in book value $\left[\left(A_t^+ - R_t^+\right) - \left(A_{t-1}^+ - R_{t-1}^+\right)\right]$ and the dividend payments D_t has to exceed $y \cdot \left(A_t^- - A_{t-1}^+\right)$.

As long as the so-called reserve quota $x_t = \dfrac{R_t}{L_t^+ + S_t^+ + B_t^+ + E_t}$ stays within a given range $[a;b]$, a target rate of interest $z > g$ is credited to the sum of the policy holders' accounts. Furthermore equity holders receive a target dividend rate of α. Thus, we get

$L_t^+ + S_t^+ + B_t^+ = (1+z)(L_t^- + S_t^- + B_t^-)$, and $A_t^+ = A_t^- - \alpha E_{t-1}$ i.e. the surplus Su_t^{PH} provided to policy holders and the dividend payments are given by

$$Su_t^{PH} = z(L_t^- + S_t^- + B_t^-) - gL_t^- \quad \text{and} \quad D_t = \alpha E_{t-1}. \tag{2}$$

Note that we do not specify, to which of the accounts *L*, *S* and *B* which portion of the policy holders' surplus is given, since this depends on the particular distribution mechanisms, that will be introduced in the next section.

As long as the reserve quota remains in $[a;b]$, this policy is followed. If however crediting *z* and α to policy holders and shareholders, respectively, would lead to a reserve quota below *a* or above *b*, respectively, surplus and dividends from (2) are reduced or increased by multiplying both with a constant factor $c \geq 0$ that leads to a reserve quota $x_t = a$ or $x_t = b$, respectively.

If no such factor $c \geq 0$ exists, this means that even paying no surplus and no dividends would lead to a reserve quota below a.

In this case, only the guaranteed rate of interest is provided to the policy reserves while the surplus and the bonus account remain unchanged and no dividends are paid, i.e.,

$$L_t^+ = (1+g)L_t^-, S_t^+ = S_t^-, B_t^+ = B_t^- \text{ and } A_t^+ = A_t^-.$$

This can however only be done, if it results in a reserve quota between 0 and *a*. If, however this would lead to reserves below 0, the bonus account is reduced by the amount needed to keep reserves at 0, i.e. we let $L_t^+ = (1+g)L_t^-$, $S_t^+ = S_t^-$, $A_t^+ = A_t^-$, and $B_t^+ = A_t^+ - L_t^+ - S_t^+ - E_t$ and thus R_t = 0. This is however only possible if the bonus account is big enough, i.e. $B_t^+ \geq 0$, or $A_t^+ \geq (1+g)L_t^- + S_t^+ + E_t$.

If the bonus account is not sufficient to provide the guaranteed rate of interest, i.e. if $A_t^- < (1+g)L_t^- + S_t^- + E_{t-1}$ then $L_t^+ = (1+g)L_t^-$, $S_t^+ = S_t^-$, $A_t^+ = A_t^-$, and $B_t^+ = 0$ which leads to negative reserves. Our model allows for negative hidden reserves as long as there is enough equity to back the liabilities. We consider it a shortfall if there is not enough equity, see below.

Finally, we have to check in each of the cases above, whether these rules comply with the restriction in asset valuation and the minimum participation rate. Whenever our decision rules lead to a violation of the restriction in asset valuation (compare footnote 7), i.e.

$$X_{BV} := y\left(A_t^- - A_{t-1}^+\right) - \left(\left(A_t^+ - R_t^+\right) - \left(A_{t-1}^+ - R_{t-1}^+\right) + D_t\right) > 0\ ,$$

we distribute the exceeding book value X_{BV} increasing the surplus provided to the policyholders by $\delta \cdot X_{BV}$ and the dividends by $(1-\delta)\cdot X_{BV}$.

Whenever our decision rules lead to a violation of the minimum participation rate, i.e.

$$\delta\left[\left(A_t^+ - R_t^+\right) - \left(A_{t-1}^+ - R_{t-1}^+\right) + D_t\right] - \Delta L + \Delta S + \Delta B > 0\ ,$$

we increase the surplus provided to the policyholders and decrease the surplus provided to the shareholders by this amount.

2.5 Surplus distribution mechanisms

We will analyze the impact of different surplus distribution mechanisms by considering three different model companies. We assume that all companies have the same balance sheet at the starting point of our analysis. In particular, we assume that the values of E_t, L_t, S_t and B_t are the same for each company which means that in the past the companies provided surplus in the same manner and only apply the different mechanism described below for future surplus. By x we denote the insurer's initial reserve quota which is given by

$$x = \frac{R_0}{L_0 + S_0 + B_0 + E_0} = \frac{A_0 - \left(L_0 + S_0 + B_0 + E_0\right)}{L_0 + S_0 + B_0 + E_0}\ .$$

Once the amount of surplus has been determined by the management decision rule given in section 2.4, the surplus distribution mechanism has to be specified. The range of possible mechanisms varies from country to country. The actual mechanism chosen differs from company to company. In our analyses we compare three different model companies:

Company 1: All surplus is credited to the policy reserves L_t, i.e. the policy reserves are increased by Su_t^{PH} every year. In this case, the guaranteed rate of return also applies to past surplus. Company 1 therefore promises cliquet-style guarantees. Note that this is the type of surplus distribution that leads to the highest future liabilities and to the least amount of flexibility for the company.

Company 2: All surplus is credited to the surplus account S_t, i.e. the policy reserves L_t are increased only by the guaranteed rate of interest, Su_t^{PH} is credited to S_t. This type of surplus distribution provides more flexibility for the insurer, since for the account S, the guaranteed interest is only 0%.

Company 3: Surplus is credited to the bonus account B_t, i.e. the policy reserves L_t are increased only by the guaranteed rate of interest, Su_t^{PH} is credited to B_t. This type of surplus distribution provides the greatest degree of flexibility for the company, since the bonus account will only be distributed to the individual policy holders at maturity of their contracts. In the meantime, money may be taken from this account to pay for interest rate guarantees if in some year the return on assets is not sufficient.

Note that although these three mechanisms lead to a different amount of flexibility and thus a different risk for the insurer, the guaranteed maturity value that is shown to the policy holder at outset, is the same in all cases.

2.6 Shortfall

We fix a time horizon of T years and consider it a shortfall, if at any balance sheet date t=1,2,…,T, the market value of the assets is lower than the book value of the policy holders' accounts, i.e. if $A_t^+ < L_t^+ + S_t^+ + B_t^+$.

We let the stopping time τ be the first balance sheet date, with a shortfall or $\tau = T+1$ if no shortfall occurs. Our numerical analyses in the next section will use the shortfall probability $P(\tau \le T)$ as a risk measure. In our model, there are many parameters that have an influence on this shortfall probability, in particular parameters describing the regulatory framework (the guaranteed rate of interest g, the minimum participation rate δ, the restriction in asset valuation y), parameters describing the insurance company's financial situation and management decisions (the current reserve situation x, the portion of stocks in the asset portfolio s, target dividend rate α, target rate of interest z, target range for the reserve quota $[a;b]$), capital market parameters, (drift μ and volatility σ of the asset portfolio), the considered time horizon T, the percentage ξ of the liabilities maturing every year, and the surplus distribution mechanism (model company 1, 2 or 3).

3. Analysis

If not stated otherwise we fix the following parameters for all our analyses:

We assume that the restriction in asset valuation is y = 50% and use a minimum participation rate of δ = 90% as required by German regulation. At t = 0, we assume the balance sheet to consist of 2% equity, 91.5% policy reserves and 6.5% terminal bonus fund which represents a typical balance sheet of a German life insurance company.

Furthermore we assume that the insurer aims to provide a target rate of interest of z = 5% to the policyholders accounts and a target dividend rate of α = 10% as long as the reserve quota stays within a range of $[a; b]$ = [5%; 30%].

We assume that contracts corresponding to ξ = 10% of the liabilities leave the company every year and fix the time horizon of our analysis to T = 10 years.

As a starting point, we look at shortfall probabilities as a function of the initial reserve quota and compare results for two different values of the guaranteed interest rate, g = 2.75% and g = 4%.[8] Additionally, we consider two different asset allocations by assuming a stock ratio in the portfolio of s = 10% or s = 30%, respectively. The results are displayed in Figure 2. Interestingly, results indicate that for a given set of parameters companies 1 and 2 behave almost identically. In other words, the question of whether the guaranteed rate of return or just a guaranteed rate of 0% is promised on past surplus does not make a major difference under these conditions[9]. For

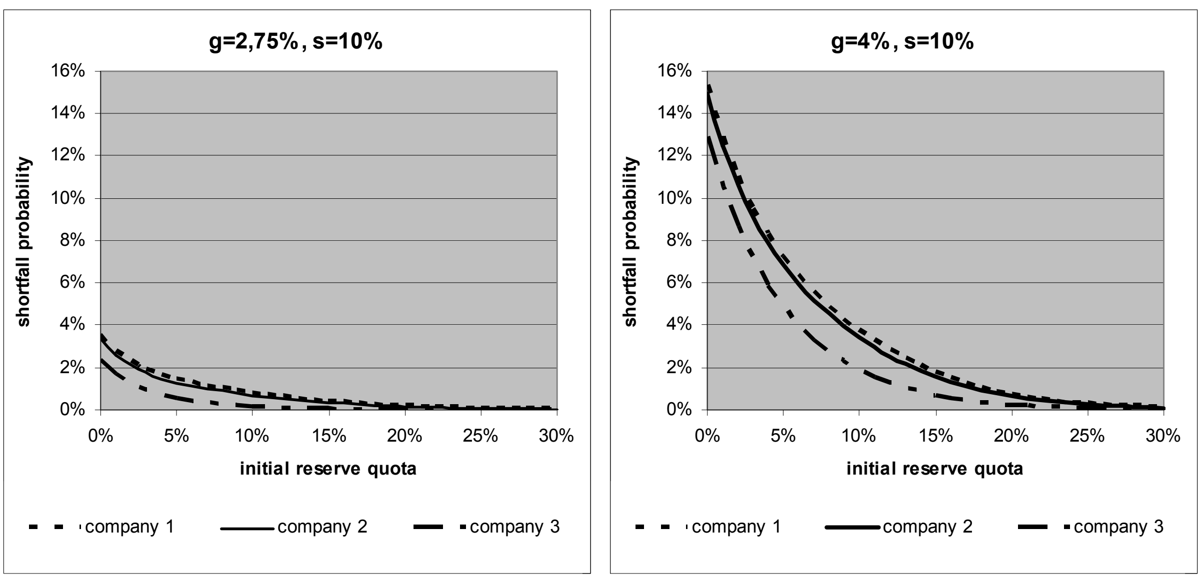
[8] The current maximum guaranteed rate for new business in the German market is 2.75%. There are still many older contracts in force that have been sold with higher rates up to 4%.

[9] This also remains true for most of the sets of results described in the following. However, the difference becomes larger if the time horizon is increased.

company 3, however, outcomes differ significantly. Generally, all other things equal, company 3 faces a much lower risk of shortfall, as it has the greatest flexibility in using former surplus as "emergency funds" to maintain interest guarantees in bad years.

Also, it can be stated that, in particular when initial reserves are low, the guaranteed rate of interest as well as the stock ratio have a considerable impact on the likelihood of shortfall. Comparing the different diagrams in Figure 2 shows, that for low reserve quota levels, increasing the guaranteed rate from 2.75% to 4% causes an increase in the shortfall probability of about 15% and increasing the stock ratio from 10% to 30% causes an increase in the shortfall probability of more than 20%. Both effects diminish for higher initial reserve quotas.

Obviously, the shortfall probability decreases as initial reserves increase. However, the marginal effect of the initial reserve quota is greater for a higher interest guarantee.



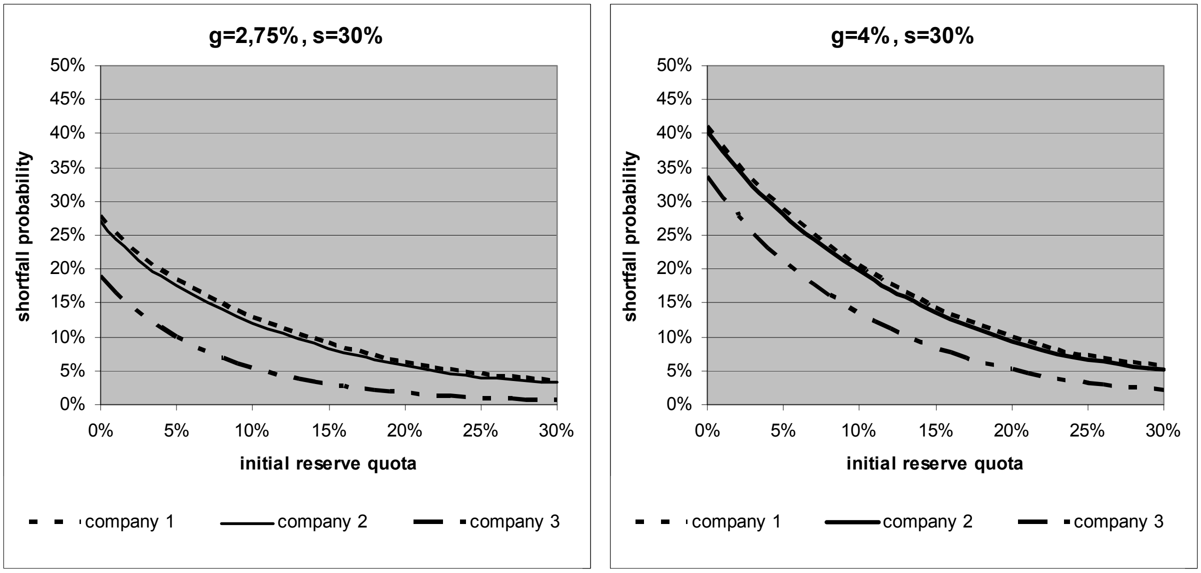


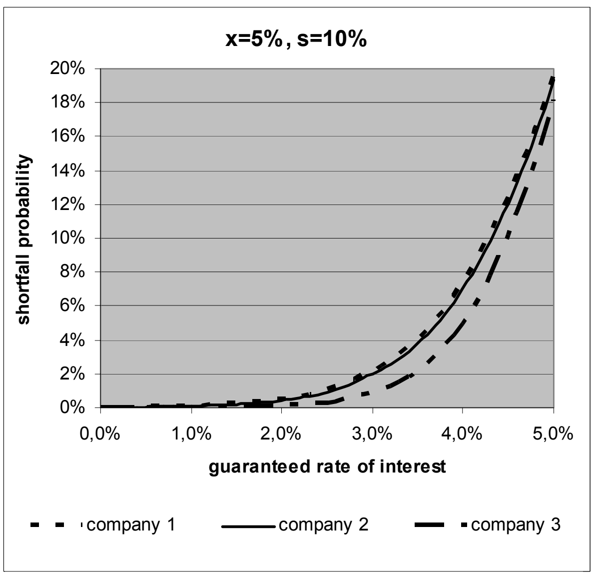
Figure 2: Shortfall probability as a function of the initial reserves for different values of the guaranteed rate of interest and for different asset allocations

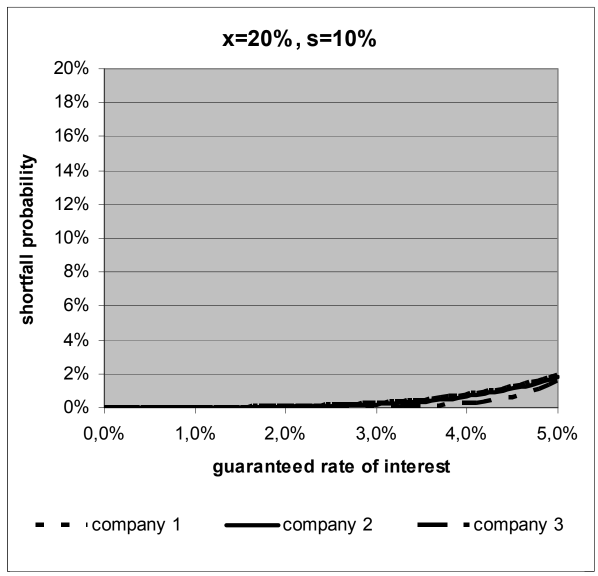
Figure 3 shows the shortfall probability as a function of the guaranteed rate of interest for different values of the initial reserve quota x and the stock portion s. Our calculations again confirm the straightforward proposition that crediting an interest rate guarantee inflicts a much higher risk on a company with a poor initial reserve level. For instance, given a reserve level of 20% and a stock portion of 30%, company 1 can offer 2.75% guaranteed interest at a shortfall probability of roughly 8%, whereas, all other things equal, a reduction in the reserve quota to 5% would bring up the shortfall risk to about 20%. At the lower reserve quota, company 3, however, would be able to provide the same guaranteed interest rate with a shortfall probability of only 10%. Again, ceteris paribus company 3 is characterized by a considerably lower shortfall risk.

Additionally, we find that not only the value of shortfall probabilities, but also the marginal impact of increasing the guaranteed interest rate is generally greater where reserves are low.

It should also be noted that for low levels of the guaranteed interest rate, the probability of shortfall tends to zero in the case of a 10% stock portion. As the guarantee is decreased, the bond portion of the insurer's asset portfolio becomes more and more likely to be sufficient to generate the minimum interest, while at the same time limiting the shortfall exposure because of the bonds' lower volatility. Whereas with a greater portion of stocks, the shortfall risk remains significantly positive for companies 1 and 2, an insurer of the company 3 type would still be able to basically avoid shortfall risk for low guaranteed rates at least in the case with higher initial reserves.

Furthermore, it must be highlighted that given a maximum tolerable shortfall probability, company 3 would at the same reserve level always be able to enter a riskier asset allocation and thus create a higher expected return for policy holders and/or provide a much higher guaranteed interest rate. For instance, consider the parameters $x = 20\%$ and $s = 30\%$. In this situation, at a shortfall probability of 5% company 1 would only be able to promise a guaranteed rate of 2%, while company 3 could offer 4%. This, of course, puts companies 1 and 2 at a significant competitive disadvantage, as for a given a-priori interest guarantee company 3 can offer contracts with a higher expected return without exposing itself to greater risk.





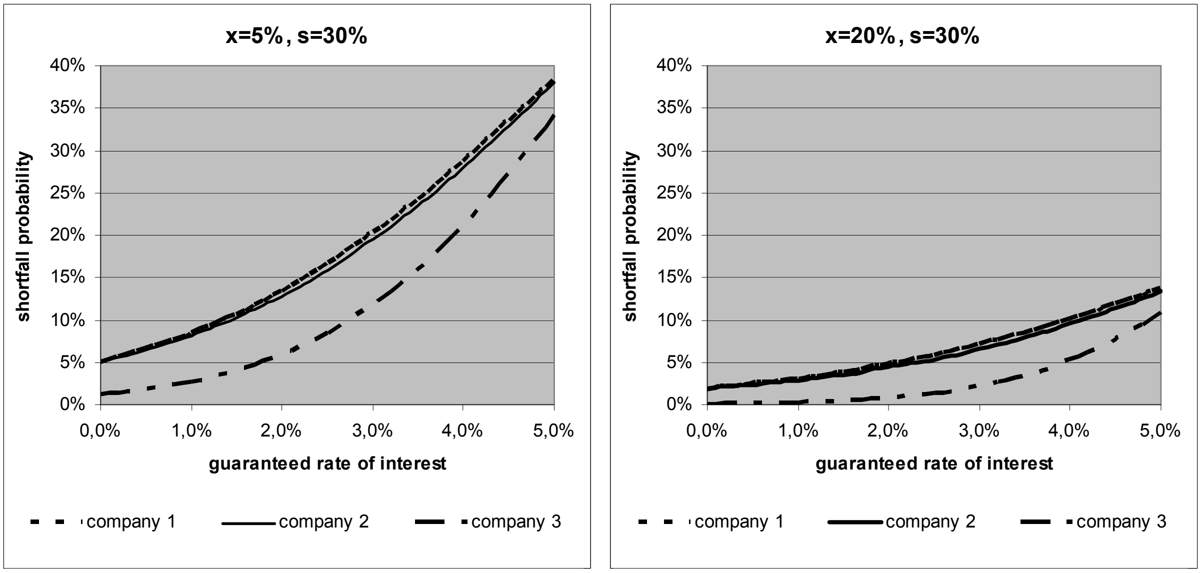


Figure 3: Shortfall probability as a function of the guaranteed rate of interest for different values of the initial reserve quota and for different asset allocations

Considering the shortfall probability as a function of the stock ratio (for $g = 2.75\%$), it can be noted that the shape of this function for the most part is not strongly affected by a change in the initial reserve situation (from $x = 5\%$ to $x = 20\%$), cf. figure 4. However, a significant difference can be observed for low stock ratios. The function seems to be convex in this area – and concave elsewhere – for all considered parameter sets. For reasons of diversification, of course the probability of shortfall decreases in the stock ratio up to a certain point, (roughly s =10%). This means that very low stock ratios (in particular: $s = 0\%$) are strictly dominated by greater levels of s which allow a higher expected return at the same risk of shortfall. This effect is particularly pronounced in a situation with low initial reserves.

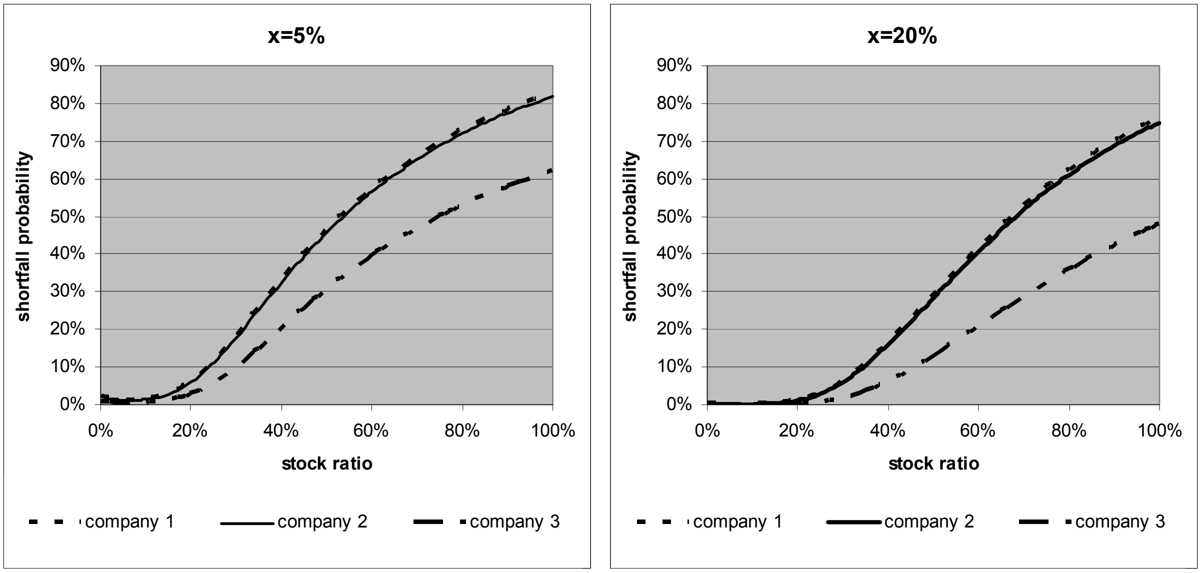


Figure 4: Shortfall probability as a function of the stock ratio s for different values of the initial reserve quota

Our standard assumption regarding the time horizon is $T = 10$. Figure 5 now shows how results react to variations of *T*. So long as the shortfall probability is positive, the effect of extending the time horizon is the more significant the smaller the value of *T*.

Again, our results also indicate that company 3 is considerably more stable than companies 1 and 2. For instance, even in the scenario with low reserves (x=5%), a guaranteed rate of interest of 4% could be provided by company 3 at a likelihood of about 15% for shortfall to occur within a time horizon of 40 years, whereas companies 1 and 2 offering the same guarantee would face the same risk of shortfall within a period of only about 17 years.

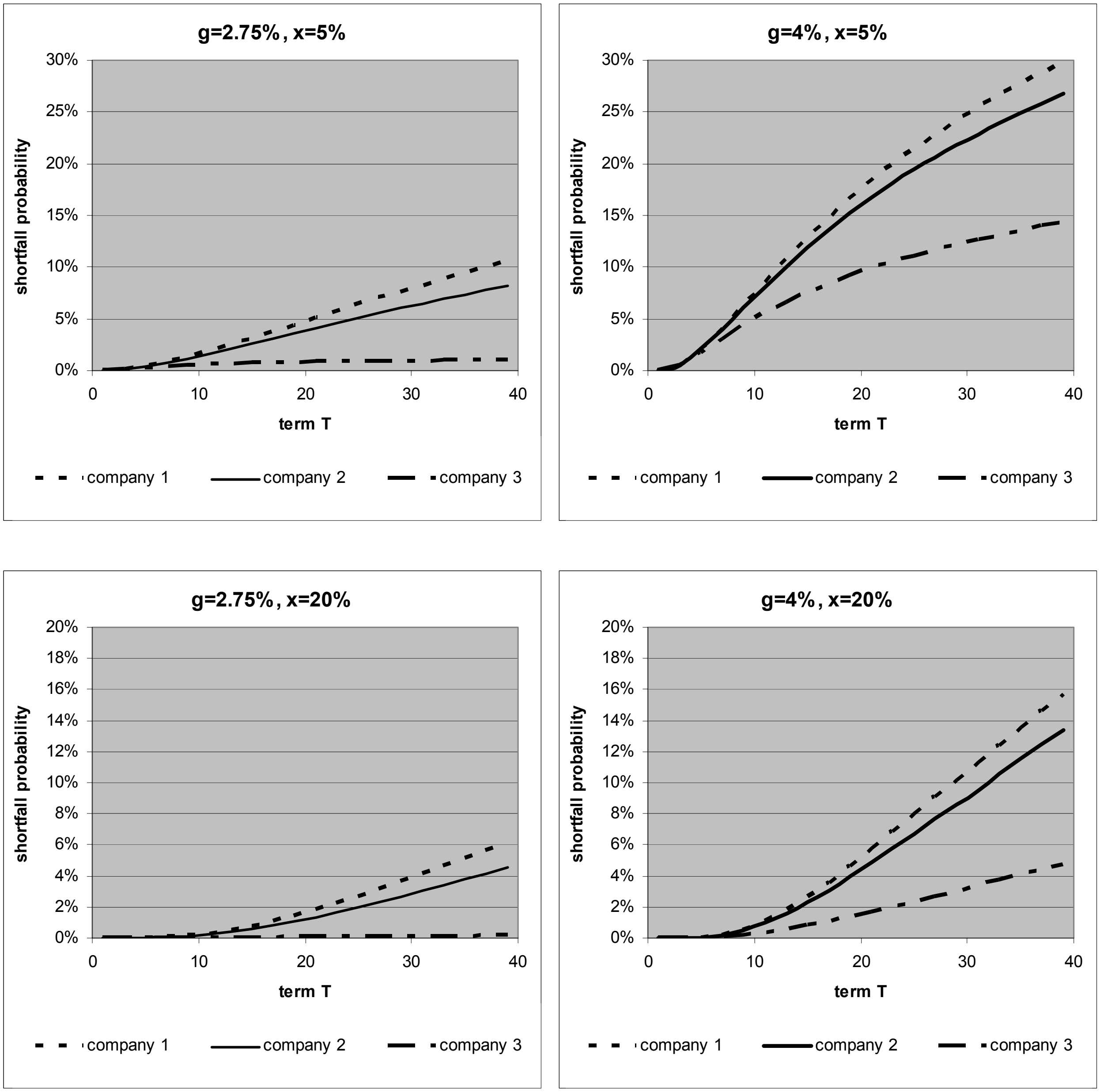
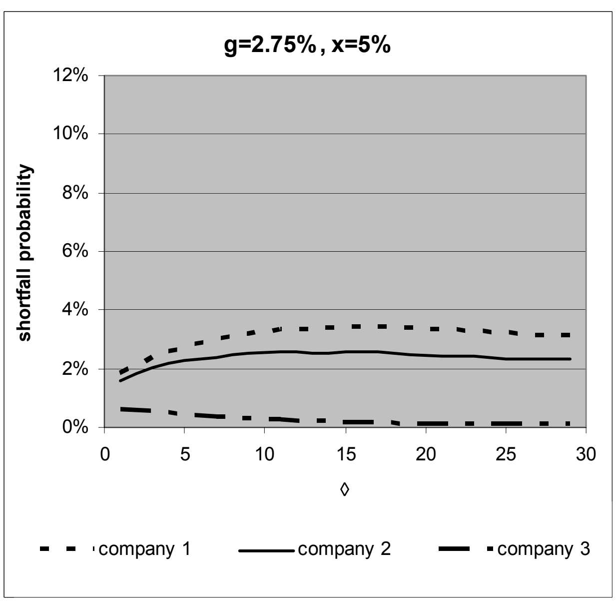
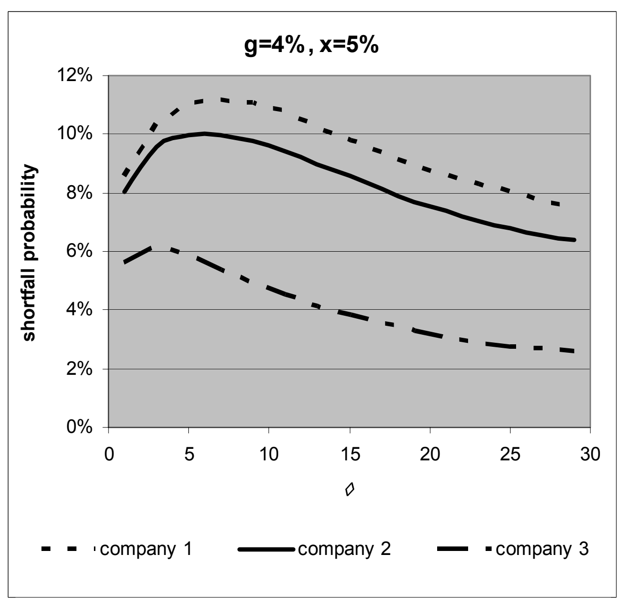


Figure 5: Shortfall probability as a function of the time horizon *T* for different values of the initial reserve quota and the guaranteed rate.

In the following, we focus on the so-called forward 10-year shortfall probabilities. By this, we mean contingent probabilities of shortfall in $[\tau, \tau + 10]$, given the fact that the insurer survived the first τ years. Figure 6 depicts these forward shortfall probabilities as a function of τ for different levels of the guaranteed rate and for different levels of the initial reserve quota. Obviously, an increase in the guaranteed rate always increases the forward shortfall risk, all other things equal. However, it does not seem to have a major impact on the shape of the function considered. Interesting observations can be made by comparing results for the two different levels of initial reserves, $x = 5\%$ and $x = 20\%$. Naturally, at $\tau = 0$, the shortfall risk is significantly lower for greater values of x. It remains lower or at least does not exceed the shortfall probability for the case of lower initial reserves, as τ varies. Remarkable, though, is the impact of increasing initial reserves on the development of shortfall risk, depending on τ: While in the low initial reserve situation the contingent shortfall risk decreases, it increases when initial reserves are at $x = 20\%$. At first, this may seem surprising. It is, however, quite intuitive, if we keep in mind the design of the rules according to which interest is credited to policyholders' accounts: Roughly, in the lower initial reserves case the insurer will credit less surplus more often as reserves tend toward the lower threshold. Thus, ceteris paribus, it is more likely that the company builds up additional reserves, eventually decreasing its forward shortfall risk, while for a company with higher initial reserves a tendency of giving high surplus and thus reducing its reserves seems to prevail for some time. While this may seem to be a competitive disadvantage for a company with greater initial reserves, we have to keep in mind that this company would be more successful in providing the target interest rate, such that in the market higher reserves signal greater stability and ultimately the better product. This said, though, it also needs to be noted that as the time horizon expands, both companies' forward shortfall risks tend towards the same value, all other things equal, which suggests that there is an equilibrium reserve level (and a corresponding default risk). Note that the graph for this level chosen as the <u>initial</u> reserve level should be entirely flat.





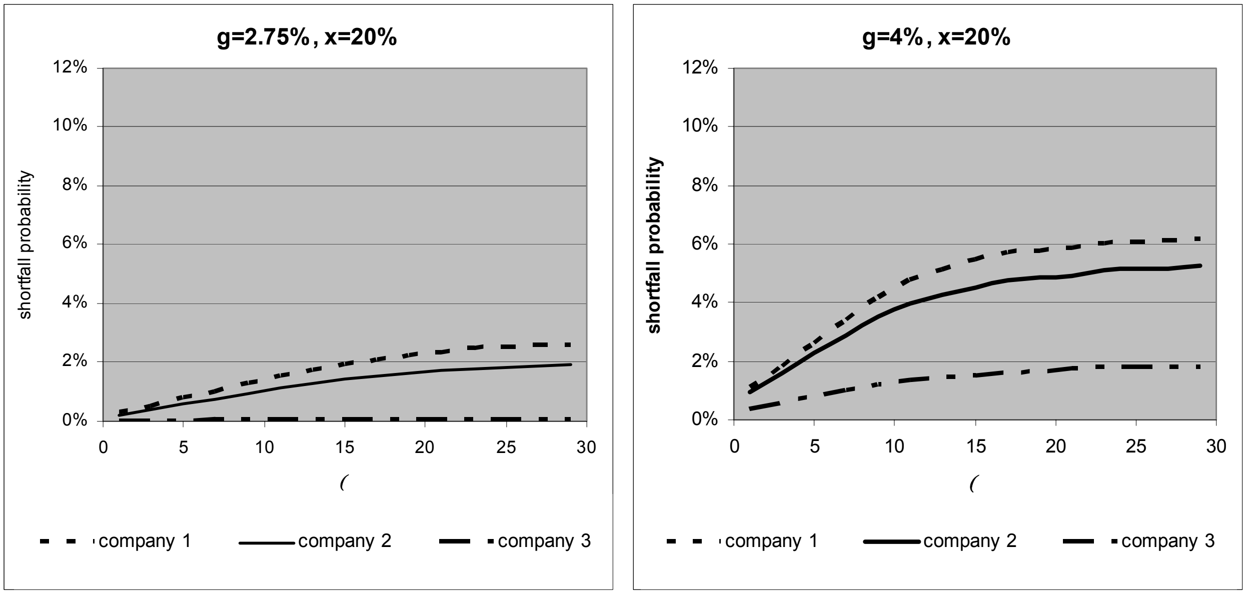


Figure 6: Forward 10-year shortfall probability as a function of the starting time τ for different values of the initial reserve quota and the guaranteed rate.

4. Conclusions

This paper analyzes the impact of different surplus distribution mechanisms on the risk exposure of a life insurance company selling with profit life insurance policies with a cliquet-style interest rate guarantee. We consider three different types of distribution mechanism: A mechanism, where the guaranteed interest rate also applies to surplus that has been credited in the past, a slightly less restrictive mechanism in which the a guaranteed rate of interest of 0% applies to past surplus, and a third mechanism that allows for the company to use former surplus in order to smoothen underperformance in "bad" years.

Our analysis suggests that under certain conditions severe regulation does not only yield suboptimal results, but also seems to be counter-productive with respect to goals that would usually be considered a fundamental purpose of regulation. A major rationale for insurance regulation typically is seen in protecting the customers by keeping insurers solvent. This is particularly important in the area of life insurance where contractual relationships are long-term and insured become major creditors whose stakes justify strong regulatory intervention.

Naturally, a strong case can also be made for protecting life insurance customers by introducing minimum interest rate guarantees and regulating surplus distribution. One must be aware, however, that inference with the way surplus is distributed, decreases an insurer's flexibility and, all other things equal, increases shortfall risk. Of course, from a social welfare standpoint this problem may be outweighed by the benefits of a minimum guarantee requirement; our study demonstrates, though, that regulators should be very careful in deciding which design of a distribution mechanism is to be enforced.

The results strongly suggest that a distribution mechanism similar to the one introduced for the third type of company mentioned above, is in many ways superior to the other two distribution mechanisms. Throughout the analysis, our representative company 3 faces ceteris paribus significantly lower shortfall risk than the other two companies.

With respect to the international marketplace, regulators should be aware that requiring "strong" guarantees puts companies at a significant competitive disadvantage relative to insurers which are subject to regulation that only requires the company 3 type surplus distribution mechanism. This is in particular true, if annual minimum participation rates are required for long term contracts.

For instance, a company of our type 3 can afford a much greater portion of stocks in its asset portfolio while maintaining the same shortfall risk, compared to type 1 and 2 companies. This means that, while subject to the same amount of risk, company 3 would be able to invest into a portfolio promising greater expected returns. On the other hand, as is straightforward but interesting to note, company 3 would also be able to, all other things equal, provide higher interest rate guarantees than companies 1 and 2 while maintaining the same shortfall risk.

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