International Association for the
Study of Insurance Economics

Études et Dossiers

Extract from

Études et Dossiers No. 302

World Risk and Insurance Economics Congress

Inaugural Conference

7 – 11 August 2005
Salt Lake City, Utah, USA

November 2005

Working Paper Series of The Geneva Association

The Geneva Association - General Secretariat - 53, route de Malagnou - CH-1208 Geneva
Tel.: +41-22-707 66 00 - Fax: +41-22-736 75 36 - secretariat@genevaassociation.org - www.genevaassociation.org

The Geneva Association Working Paper Series “Études et Dossiers” appear at irregular intervals about 10 - 12 times per year. Distribution is limited.

The “Études et Dossiers” are the working paper series of The Geneva Association. These documents present intermediary or final results of conference proceedings, special reports and research done by The Geneva Association. As they contain work in progress or summaries of conference presentations, the material must not be cited without the express consent of the author in question.

Layout & Distribution: Valéria Kozakova

Helping Workers Delay Retirement: Is it Worth it?

Marie-Eve Lachance
San Diego State University

Preliminary version - Please do not quote

June 30, 2005

Abstract

For many, delaying retirement may be a solution to saving shortfalls. However, individuals may be subject to exogenous constraints that limit their ability to postpone retirement. In this paper, we examine the increase in welfare and tax revenues that could be obtained by removing some of these constraints. We use a life-cycle model where retirement is endogenous, but also subject to exogenous constraints. We conclude that it can be interesting to invest in measures that help workers delay retirement further, but not at any cost. This type of investment would be most justified for those with little savings, high earnings, who like to work, and face retirement constraints at early ages.

1. Introduction

While it can be an ambiguous task to define retirement savings adequacy, it is often suggested that workers do not save enough for retirement. For instance, Au, Mitchell, and Phillips (2005) found that 58% of the Health and Retirement Study (HRS) respondents would experience a saving shortfall if they retired at age 65. To address this problem, there are essentially four generic strategies that can be used: 1) saving more during the working years, 2) improving the return on savings, 3) working longer, and 4) using government transfers. A lot of attention has been given recently to the first two strategies by researchers who promoted better decision-making in 401(k) plans (e.g. Choi, Laibson, Madrian, and Metrick (2004)). This paper takes a different avenue and examines the third strategy, which is to help workers delay retirement. This strategy is particularly relevant in a defined contribution plan context where lifelong savings can be jeopardized by a market downturn. It is also motivated by a recent stream of empirical and theoretical research that has examined the relation between wealth outcomes and retirement decisions.

Indeed, the recent rally in the U.S. stock market and the availability of the HRS data have made it easier to study this relation. Gustman and Steinmeier (2002), Sevak (2002), Khitatrakun (2002), and Coronado and Perozek (2003) all found that the late nineties market boom lead to early retirement. There is also evidence that this trend reversed after the market bust, although this evidence is more mixed. Eschtruth and Gemus (2002) and Kezdi and Sevak (2004) found an increase in the labor force participation of older workers between 2000 and 2003. However, Coile and Levine (2004) suggested that these aggregate results could hardly be explained by market fluctuations given the limited stock holdings of many households. Abstracting from market fluctuations, Au, Mitchell, and Phillips (2005) found a statistically significant positive relation between saving shortfalls and the probability of working longer, although the effect is not quantitatively large.

On the theoretical front, a series of parallel studies by Dybvig and Liu (2005), Farhi and Panageas (2005), and Lachance (2003, 2004) have shown how to solve a life-cycle model of optimal consumption and portfolio choices featuring an endogenous retirement date.[1] All these models predict that it is optimal to delay retirement (if possible) when savings are insufficient. The particular wealth threshold required to retire is a function of the individual's characteristics. Note that the emphasis of these papers was to show how including endogenous retirement affected the predictions obtained with traditional life-cycle models with a fixed retirement date. By contrast, here we suggest that these models can also be exploited to answer completely new research questions.

Specifically, we use a variation of these models to gain insight relatively to

[1] The concept of endogenous retirement has also been studied in the labor economics literature, but typically in a defined benefit plan context without portfolio choices. Note that Liu and Neis (2002) include portfolio choices in their model, but their definition of endogenous retirement is different from the one used in this paper and is closer to endogenous labor supply. Wooley (2004) includes endogenous retirement and portfolio choices in a model with incomplete markets and thus, only provides a numerical solution to the problem.

the strategy of delaying retirement to offset saving shortfalls. While it is fairly obvious that having the option to work longer can increase welfare, it is not as obvious that workers are able or willing to delay retirement. Indeed, the empirical results mentioned above seem to indicate that workers did not delay retirement as much as expected after the recent market bust. Therefore, our model includes preferences for leisure and exogenous retirement constraints. While in practice these exogenous constraints can take many forms (e.g. health shocks, lack of demand for older workers, etc.), we model them very simply in terms of an age $R^{\max}$ by which the worker must be retired. In this paper, we suggest that welfare gains could be generated by measures that would push $R^{\max}$ forward.[2]

In particular, our model is used to compute the welfare gains that would be generated if the exogenous retirement constraint $R^{\max}$ was delayed by one year. We find a wide range of results depending on the individual's situation and characteristics. With our base case parameters, we show that pushing $R^{\max}$ by one year is worth \$10,336 for an individual who likes to work. On the other hand, this result would be only \$1,330 for an individual who relatively dislikes to work. We also find that this strategy is particularly valuable for those who are "earnings rich" but "savings poor". Another finding is that it is more efficient to start by extending the exogenous retirement constraints at relatively younger ages (e.g. from age 62 to 63 rather than from age 68 to 69). Note that increasing earnings would also benefit governments through increased tax revenues. With our base case parameters, we find that this increase would be \$4,059 when individuals like to work and \$1,455 when they do not.

These results highlight the importance of taking into account the utility of leisure and the concept of endogenous retirement when evaluating measures where the retirement decision may be affected. Unfortunately, we know relatively little about the relative preferences for leisure and consumption at older ages and it is difficult to calibrate this type of model. Our analysis has also a few other limitations. Since we do not know how much it would cost to help workers delay retirement further, it is difficult to provide a definitive answer to the question "Is it worth it?" In addition, it is possible that the additional earnings generated by older workers come at the expense of reduced earnings for younger workers. Thus, it is not clear to which extent this substitution effect would affect our results.

[2] Note that this paper's focus is mostly on welfare gains derived from a life-cycle model and we leave for future research other interesting issues such as how to design and cost measures that would help delay $R^{\max}$. Nevertheless, one application of our results is to help guide the design of such eventual measures by identifying the groups who will benefit the most from them.

Despite these limitations, it seems worthwhile to study the question of investing to give workers the option to delay retirement further. In a context where life expectancy increases and retirement programs experience financing difficulties, increasing the labor force participation of older workers may seem a natural solution.

The paper is structured as follows. Section 2 presents the model and explains how it is calibrated for the illustrations. Section 3 describes the welfare gains that would be generated if $R^{\max}$ was delayed by one year and analyzes the comparative statics associated with these gains. Section 4 discusses how the model's other outputs would be affected by a change in $R^{\max}$ and Section 5 concludes.

2. Model and Assumptions

To measure the welfare gains that can be generated by delaying $R^{\max}$, we adapt a continuous time life-cycle model developed in Lachance (2004). As usual, the individual makes optimal decisions in terms of consumption and portfolio choices. He can also select his retirement date τ such that his utility his maximized. However, he may also be constrained to retire at a deterministic age $R^{\max}$ exogenously given. This variable is individual-specific and is meant to approximate the effect of various factors such as health, skills, demand for older workers, etc. Since modeling the retirement problem is central in this paper, we start by discussing some of the difficulties inherent to this task.

2.1. Retirement Modeling Issues

What is a realistic model of retirement? Unfortunately, there is no single answer to that question. Since each individual may face a different problem when it comes to retirement, a model that is realistic for a given individual may be quite unrealistic when applied to somebody else. One source of complexity arises from the fact that retirement itself can take many forms: from partial to total and from temporary to permanent. In addition, retirement income may come from different combinations of Social Security benefits, defined benefit (DB) plans, defined contribution (DC) plans, private savings, etc. These varied sources of retirement income may require different decisions, such as having the possibility to choose between various investments. They can also provide different retirement incentives, e.g. in the form of early retirement subsidies with DB plans.

Nevertheless, it is possible to accommodate many retirement scenarios with a fairly simple life-cycle model. With our model structure, the individual is endowed with a given amount of savings at the beginning of the problem. He then decides how much to save (i.e. consume) and how to invest these savings. All retirement income is derived from these accumulated savings and there are no other incentives to retire at a particular age. Note that it is often possible to include implicitly a DB or a DC plan in that structure. If the individual has a DC plan, we can simply augment his earnings with the employer's contribution to the DC plan. He can then save on his own the difference between his optimal level of savings and the contribution made to his DC account. Similarly, our model can easily incorporate a DB plan if its benefits are actuarially adjusted for early/late retirement.[3] In that case, we can simply add the present value of these benefits to the individual's initial savings. While the individual cannot invest this portion of his wealth directly, he can replicate the optimal portfolio strategy by leveraging his other investments.

We also adopt a straightforward definition of retirement: we assume that the individual is employed full time until he leaves the workforce completely and permanently. The alternative would be to assume that the individual can choose his quantity of labor supply in every period and that he can re-enter the workforce freely and without cost. One motivation for our modeling choice is that most recent life-cycle models with portfolio choice and endogenous retirement have adopted this definition. Another argument is that it is much easier to define a concept of endogenous/exogenous retirement in that setup. Finally, this choice seems to be supported by recent empirical evidence by Kezdi and Sevak (2004) and Coile and Levine (2004) suggesting that retirement is an absorbing state (for either supply or demand reasons). Both studies found that the re-entry rates of retirees were not significantly higher after the recent market bust period.

2.2. Economic and Demographic Assumptions

Below, we outline briefly our model and its parameters. We also present a base case set of assumptions that is used to produce next section's numerical illustrations. Note that these variables are summarized in Table 1. Since our focus is on workers nearing retirement, we consider the case of a worker who is age $t_0 = 55$

[3] The presence of early retirement subsidies could affect our analysis if they make it suboptimal for workers to delay retirement. While it would not be difficult to include these subsidies in our model, the incentives they provide are very program-specific. Thus, we have decided to exclude them from the analysis because it is difficult to use them to produce general results.

years old at the beginning of the problem. He can live up to age $T = 100$ and his conditional probability of surviving from age t to age s is given by the function $p_{t,s}$. This function is constructed with 2000 data from the National Center for Health Statistics (NCHS) to obtain a realistic unisex mortality table.

At the beginning of the problem, the individual has initial wealth w_0 and earnings y_0. Studies such as Campbell, Cocco, Gomes, and Maenhout (2000) have reported that labor income tends to decline after age 55 and we assume that earnings grow at a negative rate of $g = -1\%$ per year. For the base case scenario, we set $w_0 = \$200,000$ and $y_0 = \$25,000$. These values were chosen to mirror data from the HRS as reported by Au, Mitchell, and Phillips (2005). They correspond to median values from the third quintile of the wealth and earnings distributions for singles (around age 56). Note that the wealth figure includes the present value of Social Security benefits, retirement pension wealth, and net home equity.

Our model also includes a simplified version of Social Security and income taxes. We denote the Social Security tax rate by $\tau^{S.S.}$ and set it equal to its current level of 12.4% of earnings in the U.S.. The income tax rate is denoted by $\tau^{inc.}$ and it is derived from the tax tables given in the 2004 Internal Revenue Service 1040 form. For earnings of $y_0 = \$12,500$, $\$25,000$, and $\$50,000$ respectively, this non-linear tax rate is 7.5%, 11.2%, and 15.8%. Note that we assume that savings are tax-deductible and investment income is tax-exempt. However, the individual is taxed when he withdraws from his savings to consume. Thus, we effectively assume that the individual his taxed on his consumption.

The individual can allocate his wealth w between two assets, one risky and one risk-free. The proportion of his financial wealth invested in the risky asset is represented by a process $\{\alpha_t, t_0 \leq t < T\}$. This process is not constrained and there are no limits on borrowing and short selling. The risk-free asset earns return r and the risky asset return is given by the differential equation

$$\frac{dP_t}{P_t} = \mu dt + \sigma d\omega_t \tag{1}$$

where $0 < r < \mu < \infty$, $0 < \sigma < \infty$, $\{(\omega_t, \mathcal{F}_t), t_0 \leq t \leq T\}$ is a standard Wiener process on $\{\Omega, \mathcal{F}, P\}$, and $\{\mathcal{F}_t, t_0 \leq t \leq T\}$ is a non-decreasing, right-continuous family of σ-algebras. For our illustrations we assume that $r = 2\%$, $\mu = 6\%$, and $\sigma = 20\%$.

At retirement, we assume that the individual annuitizes his wealth. Annuity prices correspond to an annuity factor

$$a_t = \int_t^T e^{-r(s-t)} p_{t,s} ds \tag{2}$$

multiplied by a loading factor $(1+k)$. Thus, if the individual retires at time t with wealth w_t, his annuity payments are $w_t/a_t(1+k)$.[4] Note that we do not assume that the retiree has to consume this amount exactly in every period. Rather, we assume that he can decide to save some of these payments to consume later if this increases his utility. In our base case scenario, we assume that annuities are fairly priced and set $k = 0$. In the next section, we also consider the cases where annuitization is either not allowed or only possible with loaded prices.

2.3. Preferences

The individual discounts his future utility at a rate β. This utility is derived from a consumption process $\{c_t, t_0 \leq t < T\}$ and a leisure process $\{l_t, t_0 \leq t < T\}$ (both finite). We model the joint utility of leisure and consumption with a type of function commonly encountered in the life-cycle literature, which is $u(c,l) = (c \cdot l^a)^{1-\gamma}/(1-\gamma)$ with $\gamma > 1$ and $a > 0$. (Note: The parameter γ is the utility function's coefficient of relative risk aversion.) Since we assume that the individual works full time and then retires completely, we only need to consider two values of l. Let l^e and l^r (with $l^e \leq l^r$) denote respectively the quantities of leisure during the employment and retirement periods. By normalizing l^e to one, the utility functions in the employment and retirement periods become

$$u_e(c,1) = \frac{c^{1-\gamma}}{1-\gamma}, \quad c > 0, \tag{3}$$

$$u_r(c,l_r) = \frac{(c \cdot (l_r)^a)^{1-\gamma}}{1-\gamma} = \frac{(c/F)^{1-\gamma}}{1-\gamma}. \tag{4}$$

In order to express the taste for leisure in a more intuitive manner, we have substituted the parameter $0 < F \leq 1$ to $(l_r)^{-a}$ in equation (4). This parameter F corresponds to the (reduced) fraction of consumption that the individual would be willing to accept in exchange of not having to work. The lower the value of F, the greater the utility cost of having to work longer.

[4] To produce the results of the next section, the assumption regarding the timing of the annuitization was slightly modified. To simplify the exposition in the text, this discussion is left for the appendix. Note that equation (6) below has to be adjusted to take this into account.

It remains to calibrate the parameters γ and F of our utility function. For illustration purposes, the finance literature typically uses values of γ between 1 and 5. Accordingly, we select a value in that range and set $\gamma = 3$. Unfortunately, we do not have a range of typical values for the parameter F. As a result, we chose to illustrate our results for two values of F that yield different predictions. The case $F = 0.35$ represents an individual who dislikes work and would be willing to sacrifice some consumption in order to retire before age 65. The case $F = 0.70$ represents a worker who do not mind working as much. Under our base case assumptions, this individual will work as long as he can in most scenarios.

2.4. Optimization Problem

Using the assumptions defined above, we can now write the individual's optimization problem. Since the problem's parameters change after retirement, we give the pre- and post- retirement problems separately. The functions $W(t, w_t, R^{\max})$ and $R(t, w_t)$ are used to denote the amount of utility that the individual generates from wealth w_t when he is respectively working and retired. Letting $b_t = \int_t^T e^{-r(s-t)} ds$, the post-retirement optimization problem is given by:

$$R(t, w_t) = \sup_{(\alpha,c)} E\left[\int_t^T e^{-\beta(s-t)} p_{t,s} \left(\frac{c}{F}\right)^{1-\gamma} \frac{1}{1-\gamma} ds\right], \quad t \geq \tau, \tag{5}$$

$$s.t.\ \ dw_s = \left[w_s\left(\alpha_s(\mu - r) + r\right) - c_s \frac{a_\tau(1+k)}{b_\tau(1-\tau^{inc.})}\right] ds + w_s \alpha_s \sigma d\omega_s, \tag{6}$$

and for $t_0 \leq t < \tau$, the pre-retirement optimization problem is given by:

$$W(t, w_t, R^{\max}) = \sup_{(\alpha,c)} E\left[\int_t^\tau e^{-\beta(s-t)} p_{t,s} \frac{c^{1-\gamma}}{1-\gamma} ds + e^{-\beta(\tau - t)} p_{\tau,s} R(\tau, w_\tau)\right], \tag{7}$$

$$s.t.\ \ dw_s = \left[\begin{array}{c} w_s\left(\alpha_s(\mu - r) + r\right) \\ +y_s(1-\tau^{S.S.}) - \frac{c_s}{(1-\tau^{inc.})} \end{array}\right] ds + w_s \alpha_s \sigma d\omega_s, \quad w_{t_0} = w_0, \tag{8}$$

$$\tau = R^{\max} \wedge \inf\left\{s \geq t : W(s, w_s, R^{\max}) = R(s, w_s)\right\}. \tag{9}$$

Many features of the problems given in equations (5)-(6) and (7)-(9) are similar to the ones commonly found in life-cycle models, with a few exceptions. These are taxes, annuitization, and the retirement date. The appendix discusses how these assumptions translate into equations (5)-(9). The appendix also presents the solution to this optimization problem and explains how it can be evaluated numerically with an iterative technique.

3. Analysis of Welfare Gains

In this section, we analyze the welfare gains that would be generated if the exogenous retirement constraint $R^{\max}$ was extended by one year. Essentially, this figure indicates how much investment could be justified to push $R^{\max}$ forward. While the objective of this paper is not to detail the form that these investments should take, we should just mention that they could come either from an individual or from a government that wants to increase the welfare of its taxpayers. For example, both an individual and his government could invest in a training program.

More formally, let WG denote the amount of wealth that a worker would be willing to sacrifice in order to delay the exogenous retirement constraint from $R^{\max}$ to $R^{\max}+1$. This value would equate the individual's initial expected utility with the one computed with $R^{\max}+1$ and reduced wealth $w_t - WG$. Analytically, WG can be obtained from the following relation:

$$W(t_0, w_{t_0}, R^{\max}) = W(t_0, w_{t_0} - WG, R^{\max}+1). \tag{10}$$

The formula for WG is given in equation (26) in the appendix . In a nutshell, WG is equal to the present value of the additional labor income that is generated from $R^{\max}$ to $R^{\max}+1$. This amount is then adjusted by the probability that the worker is still employed during that period. It is also reduced to take into account that working longer entails a leisure utility cost. Table 2 presents the values of WG for the base case parameters and Table 3 illustrates how these results vary with various changes in assumptions.

3.1. Work/Retirement Preferences Effect

All our welfare gains results in Tables 2 and 3 are presented for the cases $F = 0.35$ and $F = 0.70$. Recall that the scenario with $F = 0.35$ represents an individual who dislikes work and who would be willing to sacrifice consumption in order to retire early. With our base case parameters, this worker would expect to retire at age 61.4 (see Table 4). Our results also show that when wealth is high relatively to earnings (e.g. when $w_0 = \$400,000$ and $y_0 = \$12,500$), it is optimal for this worker to retire immediately. By contrast, the worker with $F = 0.70$ does not mind as much the extra work and he expects to remain employed as long as he can, i.e. until age 65 when $R^{\max} = 65$.

Table 2 shows that our results can be greatly affected by the value of F. For the base case scenario with initial wealth of \$200,000 and earnings of \$25,000, the welfare gains are \$1,330 when $F = 0.35$. This is relatively low when compared to the more substantial gains of \$10,336 obtained with $F = 0.70$. Two factors explain this difference. First, since the individual with $F = 0.35$ expects to retire at age 61.4, he is less likely to exercise the option to delay retirement from age 65 to 66. Second, this additional year of work is much more costly to him in terms of foregone utility of leisure (or disutility of work).

Panel A of Table 3 shows how these results would be affected if the more extreme assumptions of $F = 0.25$ and $F = 1$ were used. For the case $F = 0.25$, it is almost always optimal to retire immediately. This scenario is less relevant for our analysis and indicates that we should focus on values of F greater than 0.25. When $F = 1$, the welfare gain increases to \$15,983 with the base case parameters. This figure is an upper bound for our results and it represents the discounted value of the additional earnings between ages 65 and 66 (without any adjustment for leisure utility costs). Since the individual is expected to work between ages 65 and 66 in both the $F = 0.70$ and $F = 1$ scenarios, we can estimate the leisure utility cost to be $\$15,983 - \$10,336 = \$5,647$ in the scenario where $F = 0.70$.

These results lead us to a few observations. First, when we study problems where individuals have to work longer, it is important to consider that 1) there is a utility cost associated with the additional work, and 2) the possibility of endogenous early retirement has to be taken into account. Unfortunately, we know relatively little about the distribution of the values of F in the population. Thus, we can compute the welfare gain for a given value of F, but we cannot evaluate the magnitude of these welfare gains at an aggregate level.

From a more practical perspective, our results indicate that investments made to help individuals remain longer in the workforce should target first those who like to work or have relatively satisfying jobs. For example, if we were to consider retraining older workers for jobs that are undesirable, the effect would be similar to what we obtained for the case $F = 0.35$. Even if we were able to design programs with costs lower than \$1,330, the absolute level of welfare gains would be fairly small. While it can be interesting to invest in order to give workers the option to work longer, we conclude that it should not be done at any cost.

3.2. Wealth Effect

It is quite intuitive to think that those with lower savings should value more the option to work longer. Table 2 shows that it is indeed the case by illustrating how halving wealth to \$100,000 and doubling it to \$400,000 affects results. For the base case scenario and $F = 0.35$, increasing wealth to \$400,000 makes it optimal to retire immediately. On the other hand, decreasing wealth from \$200,000 to \$100,000 increases considerably the welfare gain by \$2,815. Even if this individual does not like to work, he expects to remain employed until age 64.8 because his savings are quite behind schedule. Thus, the increase in welfare is due in part to the increase in the probability of exercising the option to delay retirement. Still, the welfare gain is lower than in the case with $F = 0.70$ because it is more costly for this individual to delay retirement.

Turning to the case with $F = 0.70$, we can see that this individual's welfare gain increases by \$1,451 when wealth is halved. By contrast, the gain decreases by \$2,900 when wealth doubles. In this case, the results are not explained by a change in the probability of exercising the option to delay retirement. Rather, they reflect the fact that individuals with lower levels of wealth have relatively more to gain from an additional dollar of earnings (due to the assumption that the marginal utility of consumption is decreasing).

Wealth effects can also be produced by changing the annuitization assumptions. From a mathematical perspective, allowing for annuities at fair prices is equivalent to increasing wealth. This is illustrated in Panel B of Table 3 which considers the case without annuitization. Since no annuitization is equivalent to less wealth, the welfare gains increase by \$978 (for $F = 0.35$) and \$792 (for $F = 0.70$). The same effect can be accomplished by allowing for annuitization at loaded prices.[5] For example, Panel C of Table 3 shows that for a loading factor of 30%, the welfare gains increase by \$344 (for $F = 0.35$) and by \$313 (for $F = 0.70$).

These results can also help us predict what would happen if the individual experienced a sudden drop in wealth. That could be the result of poor market performance or a cut in employer or Social Security retirement benefits. Since workers are more and more exposed to market fluctuations through individual retirement accounts, this situation is increasingly likely. In addition, many Social Security programs and defined benefit plans are currently experiencing financial

[5] If the loading increases too much, annuitizing would actually lead to lower wealth than no annuitization.

difficulty. Both factors would make it particularly interesting to invest in measures that help workers delay retirement if they have insufficient savings.

3.3. Earnings Effect

Besides our assumption for the utility of leisure, the most influential parameter in our model seems to be the level of initial earnings. To illustrate this, Table 2 also presents the results for the cases where earnings are halved to \$12,500 and doubled to \$50,000. The intuition here is that the ability to delay retirement further should be more effective when earnings are high. Indeed, for the base case scenario with $F = 0.35$, we see that doubling earnings increases considerably the welfare gains by \$6,959. Similarly, for the case $F = 0.70$ doubling earnings increases the welfare gain by \$13,238. By contrast, cutting earnings by half reduces the welfare by \$6,618.

Another way to produce similar results would be to change the earnings growth rate assumption. Recall that our base case scenario assumes that earnings grow at a (negative) real rate of $g = -1\%$ to reflect the decline in earnings at older ages. In Panel D of Table 3, we show how our results are affected when the growth rate assumption is increased to $g = 0\%$. As predicted, the welfare gains increase by respectively \$740 (for $F = 0.35$) and \$1,635 (for $F = 0.70$). In practice, the nature of an individual's job will determine whether his earnings should increase, remain level, or decrease in the last portion of his career.

One interpretation for these results is that it may not be efficient to invest in measures that help workers delay retirement if these workers' earnings are below a given threshold. In fact, any strategy that tries to increase retirement savings by exploiting human capital will not be very effective for those with limited human capital. Accordingly, individuals with low earnings potential may be better served by welfare programs than by programs that help them work longer.

We should also comment on the interaction between the wealth and earnings assumptions. In practice, it is likely that individuals with higher earnings also have higher wealth. While increasing earnings increases our welfare gains, increasing wealth makes then move in the opposite direction. According to equation (26), simultaneously doubling the wealth and earnings assumptions should double our welfare gains. This prediction can easily be verified in Tables 2 and 3.

3.4. Effect of t_0 and Initial $R^{\max}$

So far, we have arbitrarily considered that the exogenous retirement constraint was at age 65. In practice, this constraint may occur at different ages, depending on several factors such as the individual's skills and health status. In Panels E and F of Table 3, we consider how the results would be affected if $R^{\max}$ was set respectively to ages 62 and 68. For the case $F = 0.35$, the welfare gain would increase by \$231 when $R^{\max} = 62$ and would decrease by \$482 when $R^{\max} = 68$. Similarly, when $F = 0.70$ the welfare gain would increase by \$1,558 when $R^{\max} = 62$ and would decrease by \$1,276 when $R^{\max} = 68$. Part of these results can be explained by the fact that under our assumptions earnings are lower at age 68 and also they are more heavily discounted. Another factor is that the option to delay retirement is less likely to be exercised at age 68 than at age 62.

In practice, this means that it would make more sense to start by increasing labor force participation for workers in the 55-65 years group than to try to increase it for those age 65+. Although they are not explicitly in our model, there are a few arguments that could also justify investing in the "youngest" of the old workers first. One is that the disutility of working may increase with age, because of factors such as health status. Another one is that it is reasonable to think that relatively more cost and effort has to be expanded in order to retain the very oldest workers.

Note that we have also arbitrarily presented our results for a worker who starts the problem at age 55. Panels G and H of Table 3 show how our results are affected when, instead, the worker starts the problem at respectively $t_0 = 50$ and $t_0 = 60$ years old. We conclude that while the exact results change a bit, none of our previous conclusion is affected by this change in assumption.

3.5. Sensitivity of Results to Other Parameters

Panels I-M of Table 3 also present some sensitivity analysis of our results by performing the following changes in assumptions: 1) decreasing μ (the expected return on risky asset) from 6% to 4%, 2) increasing σ (the volatility of the risky asset) from 20% to 25%, 3) decreasing the discount rate β from 4% to 1%, and 4) changing the risk aversion parameter from $\gamma = 3$ to $\gamma = 2$ and $\gamma = 4$. In most cases, our results change by less than \$100. This relative insensitivity is explained by the fact that a change in these parameters affects similarly the expected utilities computed with $R^{\max} = 65$ and $R^{\max} = 66$. Thus, we observe that the conclusions derived previously in this section would not be affected by a change in the parameters μ, σ, β, and γ.

Our results could also be affected if we did not allow borrowing or short-selling, considered stochastic labor income, or add a bequest motive. Again, the intuition is the same: introducing these factors could have a significant impact on the absolute value of $W(t, x_t, R^{\max})$, but a limited impact on the relative measure WG. Readers interested in these model variations are referred to Dybvig and Liu (2005), Fahri and Panageas (2005), Lachance (2004), and Wooley (2004).

4. Impact of Delaying $R^{\max}$ on Other Model Outputs

In this section, we outline briefly how extending the exogenous retirement constraint by one year would affect the model's other outputs: expected retirement age, tax revenues, savings, and portfolio choices. These results are presented in Table 4.

4.1. Expected Time to Retirement

Panels A and B of Table 4 display the expected retirement ages when the exogenous retirement constraint is respectively 65 and 66 years old. When $R^{\max} = 65$, the worker with $F = 0.70$ expects to work until age 65 in all but one scenario. Essentially, this individual wants to work as long as he can. Thus, if we extend the maximum retirement age to 66, this individual will work one additional year and retire at age 66. By contrast, the individual with $F = 0.35$ expects to retire early at age 61.4 when $R^{\max} = 65$. If we give him the opportunity to work an extra year by setting $R^{\max} = 66$, his expected retirement age will increase only by 0.3 years. Only in poor economic scenarios will he exercise the option to work between ages 65 and 66.

Do these results imply that labor force participation should increase when $R^{\max}$ increases? Not necessarily. One limitation of the life-cycle model we use in this paper is that it does not take into account the interaction between the labor supply of young and old workers. If one individual works more, it may be at the cost of another individual working less. Thus, in aggregate, the welfare gains that we have identified so far for older workers may be offset by welfare losses for younger workers who see their labor income reduced. It is not clear to which extent older workers would substitute for younger workers. It seems reasonable to think that we should expect less substitution in tight labor markets. Also, one has to consider that if young workers would lose earnings immediately, they would also gain additional earnings when they are older. It would be interesting in future research to examine which pattern of labor supply yields more utility: more work at younger ages vs. the possibility of working longer when needed. For now, we may want to consider that delaying $R^{\max}$ still yields welfare gains, but that the results presented in Section 3 represent an upper bound for the true value of these gains.

4.2. Tax Revenues

From the point of view of governments, delaying $R^{\max}$ would yield another benefit: increasing tax revenues. Of course, as we just discussed, this is subject to limitations relatively to earnings being substituted from other workers. Still, this increase in tax revenues would be interesting because it could be viewed as a source of "self-financing" for measures helping workers delay retirement. Indeed, those who would benefit the most from these measures would also be the ones paying the most additional taxes. Then, the question is: Are these tax revenues increases important enough?

Panels C and D of Table 4 present the increases in tax revenues separately for income and Social Security taxes. In the base case scenario, total taxes would increase by $1,455 (for $F = 0.35$) and by $4,059 (for $F = 0.70$). While these results are not extraordinarily high, they could still justify investments in some low-cost programs. Note that the same observations as in Section 3 would apply. The increase in tax revenues for those who do not like to work would be relatively small. Also, little additional taxes would be generated by those with low earnings and this would be particularly true for non-linear income taxes.

4.3. Consumption and Savings

Panel E of Table 4 illustrates the effect of an increase of $R^{\max}$ on the value of retirement savings. (Here we define this value as the initial savings w_0 plus the portion of earnings that was not spent on consumption and taxes before retirement (in present value).) We show that retirement savings decrease by $3,940 (for $F = 0.35$) and by $9,087 (for $F = 0.70$). At first, this result may not seem intuitive as we may expect greater earnings to translate into greater retirement savings. One explanation for this result is that an increase in $R^{\max}$ reduces the post-retirement life expectancy and the need for retirement savings. This reduction in life expectancy will be greater for the case with $F = 0.70$ because he expects to postpone retirement by one year vs. only 0.3 year for the case with

$F = 0.35$. This argument seems to be supported by our results which show that the reduction in retirement savings is greater for the case $F = 0.70$.

With these results, we conclude that examining the amount of retirement savings in isolation is not a good benchmark to evaluate improvements in retirement savings adequacy. Rather, we should consider the effect of a measure on the individual's whole life-cycle expected utility. We also conclude that increasing $R^{\max}$ reduces the savings rate required before retirement. Accordingly, this makes it an interesting strategy for those who have difficulty saving a high enough proportion of their earnings.

4.4. Portfolio Choices

In their seminal paper, Bodie, Merton, and Samuelson (1992) have shown that introducing labor supply flexibility increases the ability to take investment risk. This problem has been extended to the flexible retirement case recently and analyzed extensively in Dybvig and Liu (2005), Fahri and Panageas (2005), and Lachance (2003, 2004). Note that when the individual is also subject to an exogenous retirement constraint, the effect of retirement flexibility may disappear if the worker is constrained to work until $R^{\max}$. Our results in Panel F of Table 4 illustrate and confirm this prediction.

If we had a model without endogenous retirement, we would see that increasing $R^{\max}$ by one year would increase the allocation to the risky asset by 2.7% (reflecting the increase in human capital). When $F = 0.35$, the individual has some retirement flexibility and Panel F of Table 4 shows that he can increase his allocation to the risky asset by 4.2%. As predicted by the theory, this increase is greater than the one obtained when retirement is exogenous. By contrast, when $F = 0.70$ the worker has little retirement flexibility and the effect of an increase in $R^{\max}$ on portfolio choice is the same as in the case where retirement is exogenous (i.e. an increase of 2.7%).

We conclude that, while increasing $R^{\max}$ has some effect on portfolio choice, we would need to increase $R^{\max}$ by several years to obtain a significant impact. This would be particularly true for those are likely to work until $R^{\max}$.

5. Conclusion

In this paper, we examined the possibility of delaying retirement as a strategy to deal with saving shortfalls. This strategy would have the advantage of increasing tax revenues and would not involve direct transfers by governments. Also, it is fairly realistic in the sense that it relies on individuals making utility-maximizing decisions regarding their labor supply.

In particular, this paper introduced an exogenous retirement constraint in a life-cycle model with endogenous retirement to answer the following question: How much welfare would be generated if it was possible (through some investment) to remove some of the exogenous retirement constraints that the individual faces? Our results reveal that the welfare gains generated by this type of investment can vary widely from one individual to another. Therefore, while these investments are potentially quite interesting, they should not be undertaken at any cost.

Accordingly, investments that give workers the option to work longer should target first the groups who would generate the highest welfare gains from this option. Our results indicate that one such group is workers who have little savings, but high earnings potential. The welfare gains would also be higher for those who like to work or face exogenous retirement constraints at relatively young ages. We note that the strategy we suggest would not be very efficient for those with low earnings. This leads us to conclude that it would be particularly important to focus on other strategies for those who cannot count on delaying retirement to make up for insufficient savings.

Our analysis has also a few limitations. While we have demonstrated that it is important to take into account the utility of leisure when studying problems where the retirement date can vary, we know relatively little about how to model and measure properly this utility. Since results are quite sensitive to this assumption, future empirical research would be helpful in terms of improving our model's calibration. Another limitation of our model is that it does not consider the interaction between various workers. Thus, the welfare gains that older workers may gain through more work could come at the expense of less income for younger workers. Finally, we did not specify how (and for what cost) we could help workers delay retirement. We leave these issues for future research.

Appendix: Discussion of Section's 2 Model and Solution

To derive the solution to the optimization problem in equations (5)-(9), we use the martingale methods developed by Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989). Accordingly, we introduce the state-price density

$$\pi_t = \exp\left\{\left(-r + \frac{\theta^2}{2}\right)(t - t_0) - \theta\left(\omega_0 + \int_{t_0}^{t} d\omega_s\right)\right\}, \qquad \theta = \frac{\mu - r}{\sigma}, \tag{11}$$

and we can rewrite the budget constraints in (6) and (8) in static form as

$$w_t = \begin{cases} E\left[\int_t^\tau \frac{\pi_s}{\pi_t}\frac{c_s}{(1-\tau^{inc})}ds + \frac{\pi_s w_\tau}{\pi_\tau} - \int_t^\tau \frac{\pi_s y_s(1-\tau^{S.S.})}{\pi_t}ds|\mathcal{F}_t\right], & t_0 \le t < \tau, \\ E\left[\int_t^T \frac{\pi_s}{\pi_t}\frac{c_s \cdot a_\tau(1+k)}{b_\tau(1-\tau^{inc})}ds|\mathcal{F}_t\right], & \tau \le t < T. \end{cases} \tag{12}$$

We then introduce a Lagrange multiplier and solve the problem with a fairly standard optimization technique. In this appendix, we present the solution to our problem, but not its derivation. Readers interested in a detailed derivation of this type of solution are referred to works by Dybvig and Liu (2005), Fahri and Panageas (2005), and Lachance (2003, 2004). Before presenting the solution to our problem, we discuss further some of the assumptions used in the paper regarding taxes, annuitization, and the retirement age.

Modeling of Taxes, Annuitization, and Retirement Age

Taxes. We take Social Security taxes into account simply by reducing earnings y_t by a factor $(1-\tau^{S.S.})$. Normally, most life-cycle models would do the same with income taxes. We take a slightly different approach to recognize that retirement savings are typically subject to a special tax treatment. Very often, contributions to retirement savings vehicles are tax-deductible and investments accumulate tax-free. However, withdrawals from these savings vehicles are taxed as ordinary income.

Mathematically, this translates into equations (6) and (8) as follows. While he is working, the individual earns labor income y_t in year t. The Social Security tax applies directly at the source and thus, the worker can potentially spend or save up to $y_t(1-\tau^{S.S.})$. Let s_t be the amount of savings in year t. Since savings are not subject to the income tax, the tax bill that year would be $\tau^{inc.}(y_t(1-\tau^{S.S.}) - s_t)$. This should be equal to $y_t(1-\tau^{S.S.}) - s_t - c_t$, i.e. the portion of earnings that was neither saved nor consumed. Accordingly, savings are $s_t = y_t(1-\tau^{S.S.}) -$

$c_t/(1-\tau^{inc.})$, the amount of taxes paid is $\tau^{inc.}c_t/(1-\tau^{inc.})$, and after tax earnings are $y_t(1-\tau^{S.S.})-\tau^{inc.}c_t/(1-\tau^{inc.})$. It is straightforward to see that equation (6) follows. Equation (8) can be obtained with the same argument and setting $y_t = 0$.

Annuitization. Without annuitization, the static budget constraint at retirement should be:

$$w_\tau = E\left[\int_\tau^T \frac{\pi_s}{\pi_\tau}\frac{c_s}{1-\tau^{inc.}}ds|\mathcal{F}_\tau\right]. \tag{13}$$

With annuitization, it is typically assumed that the individual consumes exaclty the annuity payment $w_t/a_t(1+k)$ in every period and it is not necessary to specify a post-retirement optimization problem. In that case, it is straightforward to show that $R(\tau, w_\tau) = \int_\tau^T \frac{e^{-\beta(s-\tau)}p_{\tau,s}}{1-\gamma}\left(\frac{w_\tau}{F\cdot a_\tau(1+k)}\right)^{1-\gamma}ds$. Here, we take a slightly different approach by noting that it may not be optimal to consume exactly the annuity payment in every period. Therefore, we still let the individual decide how much to consume and how to invest his wealth in every period. However, the budget constraint needs some adjustment to take into account the fact that wealth is annuitized at time τ. At that time, the individual exchanges his wealth w_τ for the discounted value of the annuity payments, which is $E\left[\int_\tau^T \frac{\pi_s}{\pi_\tau}\frac{w_\tau}{a_\tau(1+k)}ds|\mathcal{F}_\tau\right] = \frac{w_\tau b_\tau}{a_\tau(1+k)}$ where $b_\tau = \int_\tau^T e^{-r(s-\tau)}ds$. Thus, we can adjust the budget constraint in (13) by adding the gain $\frac{w_\tau b_\tau}{a_\tau(1+k)} - w_\tau$ on the left-hand side. Rewriting (13), we get

$$w_\tau = E\left[\int_\tau^T \frac{\pi_s}{\pi_\tau}\frac{c_s}{1-\tau^{inc.}}\frac{a_\tau(1+k)}{b_\tau}ds|\mathcal{F}_\tau\right]. \tag{14}$$

We should mention that there is a slight problem when we assume that annutization occurs only at the time of retirement. With our assumptions, it turns out that it would generally be optimal to annuitize as soon as possible. This is a little bit at odds with actual behavior because in practice the individual annuities market is very small, and practically inexistent before retirement. Thus, if the individual can only annuitize at retirement and retirement is delayed, he will lose some of the benefits of annuitization. This loss will be captured by WG, our formula for the welfare gain. Accordingly, if we condition annuitization on retirement, we significantly underestimate the welfare gains that the worker can generate by delaying retirement.

To alleviate this problem, one option is to assume that annuitization occurs at a fixed age. Since the latest possible value for $R^{\max}$ is age 69 in our illustrations, we assume that annuitization occurs at age 69 and modify sligthly our model in Section 2 accordingly. This approach has the advantage of removing any effect of the annuitization gains on WG, while still allowing for some post-retirement annuitization. As a result, the static budget constraint in (12) can be rewritten as

$$w_t = \begin{cases} E\left[\int_t^\tau \frac{\pi_s}{\pi_t}\frac{c_s}{(1-\tau^{inc})}ds + \frac{\pi_s w_\tau}{\pi_\tau} - \int_t^\tau \frac{\pi_s y_s(1-\tau^{S.S.})}{\pi_t}ds|\mathcal{F}_t\right], & t_0 \le t < \tau, \\ E\left[\int_t^{69} \frac{\pi_s}{\pi_t}\frac{c_s 1(t<69)}{(1-\tau^{inc})}ds + \int_{\max[t,69]}^T \frac{\pi_s}{\pi_t}\frac{c_s \cdot a_{69}(1+k)}{b_{69}(1-\tau^{inc})}ds|\mathcal{F}_t\right], & \tau \le t < T. \end{cases} \tag{15}$$

Retirement age. To compute the endogenous retirement age, we assume that the worker will not retire as long as this would result in a loss of utility. Therefore, he will retire as soon as the utility of retiring (measured by $R(t, w_t)$) is equal to the utility of continuing to work (measured by $W(t, w_t, R^{\max})$). However, he will retire at $R^{\max}$ if this endogenous retirement condition is not met by that time. Analytically, this translates into a stopping time $\tau = R^{\max} \wedge \inf\{s \ge t : W(s, w_s, R^{\max}) = R(s, w_s)\}$.

Solution to Section 2's Optimization Problem

To express the solution to the optimization problem in (5)-(9) (subject to the annuitization adjustment in (15), we introduce the functions $A(t)$, $A(t, w_t, R^{\max})$, and $Y(t, w_t, R^{\max})$. For $\tau \le t < T$, the value function $R(t, w_t)$ is given by

$$R(t, w_t) = \frac{w_t^{1-\gamma} A^\gamma(t)}{1-\gamma}, \tag{16}$$

where

$$\begin{aligned} A(t) = 1(t < 69)\left(\frac{F}{(1-\tau^{inc.})}\right)^{1-1/\gamma} \int_t^{69} \left[\frac{\exp\left\{\left(\left(0.5\frac{\theta^2}{\gamma}+r\right)(\gamma-1)+\beta\right)(s-t)\right\}}{p_{t,s}}\right]^{-1/\gamma} ds \\ + \left(\frac{F a_{69}(1+k)}{b_{69}(1-\tau^{inc.})}\right)^{1-1/\gamma} \int_{\max[t,69]}^T \left[\frac{\exp\left\{\left(\left(0.5\frac{\theta^2}{\gamma}+r\right)(\gamma-1)+\beta\right)(s-t)\right\}}{p_{t,s}}\right]^{-1/\gamma} ds. \end{aligned} \tag{17}$$

For $t_0 \le t < \tau$, the value function $W(t, w_t, R^{\max})$ is given by

$$W(t, w_t, R^{\max}) = [w_t + Y(t, w_t, R^{\max})]^{1-\gamma} \frac{A^\gamma(t, w_t, R^{\max})}{1-\gamma}, \tag{18}$$

where

$$A(t,w_t,R^{\max}) = E\left[\int_t^{\tau}\left(\frac{\pi_s}{\pi_t(1-\tau^{inc.})}\right)^{1-1/\gamma}\left(\frac{e^{\beta(s-t)}}{p_{t,s}}\right)^{-1/\gamma}ds|\mathcal{F}_t\right] + E\left[\left(\frac{\pi_\tau}{\pi_t}\right)^{1-1/\gamma}\left(\frac{e^{\beta(\tau-t)}}{p_{t,\tau}}\right)^{-1/\gamma}A(\tau)|\mathcal{F}_t\right], \tag{19}$$

$$Y(t,w_t,R^{\max}) = E\left[\int_t^{\tau}\frac{\pi_s}{\pi_t}y_s(1-\tau^{S.S.})ds|\mathcal{F}_t\right], \tag{20}$$

and

$$\tau|\mathcal{F}_t = R^{\max}\wedge\inf\left\{s\geq t : W(s,w_s,R^{\max}) = R(s,w_s)\right\}. \tag{21}$$

The budget constraint can also be rewritten as

$$w_t = \begin{cases}\left(\lambda_0\frac{\pi_t}{\pi_{t_0}}\frac{e^{\beta(t-t_0)}}{p_{t_0,t}}\right)^{-1/\gamma}A(t,w_t,R^{\max}) - Y(t,w_t,R^{\max}) & \text{for } t_0\leq t<\tau,\\ \left(\lambda_0\frac{\pi_t}{\pi_{t_0}}\frac{e^{\beta(t-t_0)}}{p_{t_0,t}}\right)^{-1/\gamma}A(t) & \text{for } \tau\leq t<T,\end{cases} \tag{22}$$

where λ_0 is the Lagrange multiplier of the problem and it satisfies the initial budget constraint

$$w_0 = \lambda_0^{-1/\gamma}A(t_0,w_{t_0},R^{\max}) - Y(t_0,w_{t_0},R^{\max}). \tag{23}$$

It also follows that we can write the optimal consumption process as

$$c^*(t,w_t,R^{\max}) = \begin{cases}\left[\lambda_0\frac{\pi_t}{\pi_{t0}}\frac{e^{\beta(t-t_0)}}{p_{t_0,t}}\frac{1}{(1-\tau^{inc.})}\right]^{-1/\gamma} & \text{for } t_0\leq t<\tau,\\ \left[\lambda_0\frac{\pi_t}{\pi_{t_0}}\frac{e^{\beta(t-t_0)}}{p_{t_0,t}}\frac{F^{1-\gamma}}{(1-\tau^{inc.})}\right]^{-1/\gamma} & \text{for } \tau\leq t<69,\\ \left[\lambda_0\frac{\pi_t}{\pi_{t_0}}\frac{e^{\beta(t-t_0)}}{p_{t_0,t}}\frac{F^{1-\gamma}a_{69}(1+k)}{b_{69}(1-\tau^{inc.})}\right]^{-1/\gamma} & \text{for } 69\leq t<T,\end{cases} \tag{24}$$

and the optimal proportion of financial wealth allocated to the risky asset as

$$\alpha^*(t,w_t,R^{\max}) = \begin{cases}\frac{\partial w_t/\partial\omega_0}{w_t\sigma} & \text{for } t_0\leq t<\tau,\\ \frac{(\mu-r)}{\sigma^2\gamma} & \text{for } \tau\leq t<T.\end{cases} \tag{25}$$

Formula for Table 2 and 3's Welfare Gains

Using equations (10) and (18), it is straightforward to find the solution to WG, the welfare gains presented in Tables 2 and 3. This solution is given by

$$WG = \underbrace{Y(t, w_t, R^{\max} + 1) - Y(t, w_t, R^{\max})}_{\geq 0} \\ + [w_t + Y(t, w_t, R^{\max})] \underbrace{\left[1 - \left(\frac{A(t, w_t, R^{\max})}{A(t, w_t, R^{\max} + 1)}\right)^{\gamma/(1-\gamma)}\right]}_{\leq 0}. \quad (26)$$

The first component of this solution represents the increase in human capital (discounted value of earnings) that follows when $R^{\max}$ is increased by one year. Note that this increase may be less than a year of earnings for those workers who would retire early under some scenarios. The second component reduces the value of these earnings to take into account the leisure cost (the effect of leisure is incorporated in the ratio $A(t, w_t, R^{\max})/A(t, w_t, R^{\max} + 1)$). We note that this reduction is more important for those with higher total wealth $w_t + Y(t, w_t, R^{\max})$ since leisure is relatively more appealing than consumption at higher levels of wealth. Also, it is quite straightforward from equation (26) to see that welfare gains double when wealth and earnings are doubled simultaneously.

Numerical Evaluation

The first step in our numerical evaluation is to use the bisection method and equation (23) to solve for λ_0. In that process, the functions $Y(t, w_t, R^{\max})$, $A(t, w_t, R^{\max})$, and $A(\tau)$ must be evaluated numerically. For that purpose, we discretize the integrals in these equations with an interval $\Delta t = 0.1$. The function $A(\tau)$ can be obtained simply by numerical integration. Computing $Y(t, w_t, R^{\max})$ and $A(t, w_t, R^{\max})$ is more complicated since we need to know the distribution of τ, the endogenous retirement date. To get around this problem, we fix a value for λ_0 and project the values of $\lambda_0 \pi_t / \pi_{t_0}$ at every node of the tree. We start by computing the value of our functions at $R^{\max}$, the last possible time. We then move back in time to complete the tree.

Specifically, it is straightforward to compute $Y(t, w_t, R^{\max}) = 0$ and $A(t, w_t, , R^{\max}) = A(R^{\max})$ at time $t = R^{\max}$. Accordingly, we can obtain w_t and $R(t, w_t)$ at every node by applying equations (18) and (16). We then move back one period in time at $t = R^{\max} - \Delta t$ and start by assuming that the individual will work between $R^{\max} - \Delta t$ and $R^{\max}$. Accordingly, we can compute iteratively the values of $Y(t, w_t, R^{\max})$, $A(t, w_t, R^{\max})$, w_t, $R(t, w_t)$, and $W(t, w_t, R^{\max})$ using the formulas given in this appendix. At every node, we can test whether it is optimal to work by comparing the values of $W(t, w_t, R^{\max})$ and $R(t, w_t)$. At the nodes where it is optimal to retire, we revise the values of $Y(t, w_t, R^{\max})$ and $A(t, w_t, R^{\max})$ by assuming that the individual does not work between $R^{\max} - \Delta t$ and $R^{\max}$. We can then move back in time another period at time $t = R^{\max} - 2\Delta t$ and repeat the same procedure. This process is repeated until time t_0. At this time, we apply the bisection method and test whether the difference $w_0 - \left[\lambda_0^{-1/\gamma} A(t_0, w_{t_0}, R^{\max}) - Y(t_0, w_{t_0}, R^{\max})\right]$ is small enough. If not, we change the value of λ_0 until we get enough precision. Once λ_0 is obtained, it is straightforward to use our trinomial tree to obtain the exact values of $Y(t_0, w_{t_0}, R^{\max})$ and $A(t_0, w_{t_0}, R^{\max})$, which allows us to generate the results of Tables 2, 3, and 4.

References

[1] Au, A., O. S. Mitchell, and J. W. R. Phillips, 2005, "Saving Shortfalls and Delayed Retirement," Michigan Retirement Research Center, Working Paper 2005-094.

[2] Bodie, Z., R. C. Merton, and W. F. Samuelson, 1992, "Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model," Journal of Economic Dynamics and Control, 16, 427-449.

[3] Campbell, J. Y., J. F. Cocco, F. J. Gomes, and P. J. Maenhout, 2000, "Investing Retirement Wealth: A Life-Cycle Model," in J.Y. Campbell and M. Feldstein, (eds.), Risk Aspects of Investment-Based Social Security Reform, University of Chicago Press.

[4] Choi, J. J., D. Laibson, B. C. Madrian, A. Metrick, 2004, "Saving for Retirement on the Path of Least Resistance," Forthcoming in E. McCaffrey and J. Slemrod, (eds.), *Behavioral Public Finance*, New York: Russell Sage Foundation.

[5] Coile, C. C. and P. B. Levine, 2004, "Bulls, Bears, and Retirement Behavior," Working Paper 10779, National Bureau of Economic Research, Cambridge, MA.

[6] Coronado, J. L. and M. Perozek, 2003, "Wealth Effects and the Consumption of Leisure: Retirement Decisions During the Stock Market Boom of the 1990s," FEDS Working Paper 2003-20, Federal Reserve Board, Washington, DC.

[7] Cox, J. C. and C. Huang, 1989, "Optimal Consumption and Portfolio Policies when Asset Prices Follow a Diffusion Process," Journal of Economic Theory, 49, 33-83.

[8] Dybvig, P. H. and H. Liu, 2005, "Lifetime Consumption and Investment: Retirement and Constrained Borrowing," Working Paper, Washington University in Saint-Louis.

[9] Eschtruth, A. D. and J. Gemus, 2002, "Are Older Workers Responding to the Bear Market," Center for Retirement Research at Boston College, Just the Facts on Retirement Issues #5.

[10] Farhi, E. and S. Panageas, 2005, "Saving and Investing for Early Retirement: A Theoretical Analysis," Working Paper, MIT and Wharton School.

[11] Gustman, A. L. and T. L. Steinmeier, 2002, "Retirement and the Stock Market Bubble," Working Paper No. 9404, National Bureau of Economic Research, Cambridge, MA.

[12] Karatzas, I., J. P. Lehoczky, and S. E. Shreve, 1987, "Optimal Portfolio and Consumption Decisions for a Small Investor on a Finite Horizon," SIAM Journal of Control and Optimization, 25, 1557-1586.

[13] Kezdi, G. And P. Sevak, 2004, "Economic Adjustment of Recent Retirees to Adverse Wealth Shocks," Michigan Retirement Research Center, Working Paper 2004-075.

[14] Khitatrakun, S., 2002, "Wealth and the Timing of Retirement," Working Paper, University of Wisconsin-Madison.

[15] Lachance, M. E., 2003, "Optimal Retirement Behavior as Retirement Looms," working paper, Wharton School.

[16] Lachance, M. E., 2004, "Managing Retirement Wealth Risk," Working Paper, City University.

[17] Liu, J. and E. Neis, 2002, "Endogenous Retirement and Portfolio Choice," Working Paper, University of California at Los Angeles.

[18] Merton, R. C., 1971, "Optimal Consumption and Portfolio Rules in a Continuous-Time Model," Journal of Economic Theory, 3, 373-413.

[19] Sevak, P., 2002, "Wealth Shocks and Retirement Timing: Evidence from the Nineties," Working Paper 2002-027, Michigan Retirement Research Center.

[20] Wooley, M., 2004, "Optimal Consumption and Investment with Labor Income Uncertainty and Endogenous Retirement," Working Paper, University of North Carolina at Chapel Hill.

Table 1. Base Case Parameters Values
(in real terms)

Variable	Symbol	Value
Risk-free rate	r	2%
Expected return on risky asset	μ	6%
Standard deviation of return on risky asset	σ	20%
Time preference discount rate	β	4%
Coefficient of relative risk aversion	γ	3
Leisure preference parameter	F	0.35 and 0.70
Age at start of problem	t_0	55
Maximum age	T	100
Exogenous retirement constraint	$R^{\max}$	65
Wealth at age 55	x_0	$200,000
Earnings at age 55	y_0	$25,000
Earnings growth rate	g	-1%
Probability of surviving from age t to s	$p_{t,s}$	Realistic table table constructed with NCHS data (unisex)
Loading on annuities	k	0%
Social Security tax rate	$\tau^{S.S.}$	12.4%
Income tax rate	$\tau^{inc.}$	7.5% for $y_0 = \$12,500$ 11.2% for $y_0 = \$25,000$ 15.8% for $y_0 = \$50,000$

Table 2. Welfare Gains when the Exogenous Retirement Constraint $R^{\max}$ is Extended by One Year (from age 65 to age 66)*

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$665	\$4,145	\$12,170	\$5,168	\$11,787	\$25,025
x_0 at	\$200,000	n/a	\$1,330	8,289	3,718	10,336	23,574
age 55	\$400,000	n/a	n/a	2,660	1,090	7,436	20,673

* Results computed with equation (26) and the assumptions given in Table 1. Entries with "n/a" indicate that it is optimal to retire immediately at age 55 for that level of wealth and earnings. The parameter F represents the individual's taste for leisure: the lower the value of F, the more appealing it is to retire early.

Table 3. Testing the Sensitivity of Table 2's Welfare Gains to a Change in Assumptions*

Panel A: $F = 0.25$ and $F = 1$ *(instead of* $F = 0.35$ and $F = 0.70$*)*

		$F = 0.25$			$F = 1$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	n/a	\$1,781	\$7,032	\$7,991	\$15,983	\$31,966
x_0 at	\$200,000	n/a	n/a	\$3,562	7,991	15,983	31,966
age 55	\$400,000	n/a	n/a	n/a	7,991	15,983	31,966

Panel B: No annuitization
(instead of annuitization with 0% loading)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$1,154	\$5,351	\$14,324	\$5,564	\$12,375	\$25,998
x_0 at	\$200,000	n/a	2,308	10,702	4,317	11,128	24,751
age 55	\$400,000	n/a	n/a	4,615	1,891	8,633	22,256

Panel C: Annuitization with 30% loading
(instead of annuitization with 0% loading)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$837	\$4,596	\$12,999	\$5,324	\$12,019	\$25,409
x_0 at	\$200,000	n/a	1,674	9,193	3,954	10,649	24,039
age 55	\$400,000	n/a	n/a	3,347	1,387	7,908	21,298

* Results computed with equation (26) and the assumptions given in Table 1 (with the exception of the change in parameter value given in each panel's title). Entries with "n/a" indicate that it is optimal to retire immediately at age 55 for that level of wealth and earnings. The parameter F represents the individual's taste for leisure: the lower the value of F, the more appealing it is to retire early.

Table 3 (Continued)

Panel D: $g = 0\%$ (instead of $g = -1\%$)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$1,035	\$5,455	\$14,924	\$5,985	\$13,422	\$28,294
x_0 at	\$200,000	n/a	2,070	10,911	4,535	11,971	26,843
age 55	\$400,000	n/a	n/a	4,141	1,743	9,069	23,942

Panel E: $R^{\max}$ extended from age 62 to age 63
(instead of from age 65 to age 66)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$780	\$5,507	\$16,012	\$5,947	\$13,548	\$28,749
x_0 at	\$200,000	n/a	1,561	11,015	4,293	11,894	27,096
age 55	\$400,000	n/a	n/a	3,122	1,187	8,587	23,788

Panel F: $R^{\max}$ extended from age 68 to age 69
(instead of from age 65 to age 66)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$424	\$3,344	\$9,649	\$4,530	\$10,333	\$21,939
x_0 at	\$200,000	n/a	848	6,688	3,259	9,060	20,666
age 55	\$400,000	n/a	n/a	1,696	1,020	6,519	18,119

Panel G: Age t_0 at start of problem is 50 years old (instead of 55)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$782	\$3,490	\$9,486	\$4,403	\$9,872	\$20,810
x_0 at	\$200,000	n/a	1,564	6,980	3,340	8,807	19,744
age 55	\$400,000	n/a	n/a	3,128	1,384	6,679	17,613

Table 3 (Continued)

Panel H: Age t_0 at start of problem is *60 years old (instead of 55)*

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		$12,500	$25,000	$50,000	$12,500	$25,000	$50,000
Wealth	$100,000	$380	$5,412	$17,331	$6,139	$14,363	$30,812
x_0 at	$200,000	n/a	761	10,824	4,053	12,277	28,726
age 55	$400,000	n/a	n/a	1,521	431	8,106	24,554

Panel I: $\mu = 4\%$ (instead of $\mu = 6\%$)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		$12,500	$25,000	$50,000	$12,500	$25,000	$50,000
Wealth	$100,000	$373	$3,988	$12,120	$5,181	$11,806	$25,057
x_0 at	$200,000	n/a	746	7,976	3,737	10,362	23,613
age 55	$400,000	n/a	n/a	1,491	913	7,474	20,724

Panel J: $\sigma = 25\%$ (instead of $\sigma = 20\%$)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		$12,500	$25,000	$50,000	$12,500	$25,000	$50,000
Wealth	$100,000	$573	$4,050	$12,134	$5,174	$11,796	$25,040
x_0 at	$200,000	n/a	1,145	8,100	3,726	10,348	23,592
age 55	$400,000	n/a	n/a	2,291	1,022	7,452	20,696

Panel K: $\beta = 1\%$ (instead of $\beta = 4\%$)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		$12,500	$25,000	$50,000	$12,500	$25,000	$50,000
Wealth	$100,000	$691	$4,134	$12,156	$5,216	$11,859	$25,144
x_0 at	$200,000	n/a	1,382	8,269	3,791	10,433	23,718
age 55	$400,000	n/a	n/a	2,764	1,202	7,581	20,865

Table 3 (Continued)

Panel L: $\gamma = 2$ (instead of $\gamma = 3$)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$700	\$4,381	\$12,169	\$5,159	\$11,773	\$25,002
x_0 at	\$200,000	n/a	1,400	8,761	3,719	10,318	23,547
age 55	\$400,000	n/a	n/a	2,799	1,268	7,438	20,636

Panel M: $\gamma = 4$ (instead of $\gamma = 3$)

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	534	\$4,141	\$12,325	\$5,185	\$11,812	\$25,065
x_0 at	\$200,000	n/a	1,069	8,281	3,742	10,369	23,623
age 55	\$400,000	n/a	n/a	2,138	1,012	7,484	20,738

Table 4. Impact on Expected Retirement Age, Tax Revenues, Savings, and Portfolio Choices when the Exogenous Retirement Constraint $R^{\max}$ is Extended by One Year (from age 65 to age 66)*

Panel A: Expected retirement age with $R^{\max} = 65$

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		$12,500	$25,000	$50,000	$12,500	$25,000	$50,000
Wealth	$100,000	61.4	64.8	65.0	65.0	65.0	65.0
x_0 at	$200,000	55.0	61.4	64.8	65.0	65.0	65.0
age 55	$400,000	55.0	55.0	61.4	63.2	65.0	65.0

Panel B: Expected retirement age with $R^{\max} = 66$

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		$12,500	$25,000	$50,000	$12,500	$25,000	$50,000
Wealth	$100,000	61.7	65.6	66.0	66.0	66.0	66.0
x_0 at	$200,000	55.0	61.7	65.6	66.0	66.0	66.0
age 55	$400,000	55.0	55.0	61.7	64.0	66.0	66.0

Panel C: Increase in income taxes

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		$12,500	$25,000	$50,000	$12,500	$25,000	$50,000
Wealth	$100,000	$216	$1,563	$4,926	$599	$1,797	$5,035
x_0 at	$200,000	n/a	649	4,387	598	1,797	5,044
age 55	$400,000	n/a	n/a	1,824	489	1,794	5,044

* Results computed with the assumptions given in Table 1. Entries with "n/a" indicate that it is optimal to retire immediately at age 55 for that level of wealth and earnings. The parameter F represents the individual's taste for leisure: the lower the value of F, the more appealing it is to retire early.

Table 4 (Continued)

Panel D: Increase in Social Security taxes

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$403	\$1,967	\$4,432	\$1,131	\$2,262	\$4,525
x_0 at	\$200,000	n/a	806	3,934	1,129	2,262	4,525
age 55	\$400,000	n/a	n/a	1,612	925	2,259	4,525

Panel E: Decrease in present value of retirement savings

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	\$1,970	\$5,112	\$7,220	\$4,545	\$5,063	\$6,071
x_0 at	\$200,000	n/a	3,940	10,225	8,555	9,087	10,125
age 55	\$400,000	n/a	n/a	7,883	13,434	17,107	18,174

Panel F: Increase in proportion of financial wealth allocated to the risky asset

		$F = 0.35$			$F = 0.70$		
		Earnings y_0 at age 55			Earnings y_0 at age 55		
		\$12,500	\$25,000	\$50,000	\$12,500	\$25,000	\$50,000
Wealth	\$100,000	4.2%	6.2%	11.5%	2.7%	5.3%	10.6%
x_0 at	\$200,000	n/a	4.2%	6.2%	1.3%	2.7%	5.3%
age 55	\$400,000	n/a	n/a	4.1%	0.8%	1.3%	2.7%