International Association for the
Study of Insurance Economics

Études et Dossiers

Extract from

Études et Dossiers No. 302

**World Risk and Insurance
Economics Congress**

Inaugural Conference

7 – 11 August 2005
Salt Lake City, Utah, USA

November 2005

Working Paper Series of
The Geneva Association

The Geneva Association - General Secretariat - 53, route de Malagnou - CH-1208 Geneva
Tel.: +41-22-707 66 00 - Fax: +41-22-736 75 36 - secretariat@genevaassociation.org - www.genevaassociation.org

The Geneva Association Working Paper Series “Études et Dossiers” appear at irregular intervals about 10 - 12 times per year. Distribution is limited.

The “Études et Dossiers” are the working paper series of The Geneva Association. These documents present intermediary or final results of conference proceedings, special reports and research done by The Geneva Association. As they contain work in progress or summaries of conference presentations, the material must not be cited without the express consent of the author in question.

Layout & Distribution: Valéria Kozakova

The Effect of an Increase in the Probability of Loss When Risk is Endogenous

by **Henri Loubergé and Richard Watt**

Abstract

Traditional economic theory of insurance is based on several simplifying assumptions, among which one of the most restrictive is that the risk against which insurance is to be purchased is entirely exogenous. Not surprisingly, the optimal purchase of, and the comparative statics of, insurance are critically dependent upon this assumption. In particular, in this paper we consider the effect of an increase in the probability of loss when the restriction to exogenous risk is relaxed.

1 Introduction and motivation

In the “traditional” or “standard” model of the demand for insurance (Mossin, 1968), the amount of risk is not a choice variable, but rather risk is fully endowed. In this model, it is quite straightforward to show that if the probability of the loss state is increased, then conditional upon the premium per unit coverage not changing, the amount of insurance will increase to compensate the risk averse insured for this increase in risk. However, this result is misleading for two reasons. Firstly, in a model of full and symmetric information, it is unreasonable to assume that the premium will not change when the probability of the loss state is increased. Indeed, since the premium for insurance will normally be calculated as an increasing function of the probability of loss (often, it is a linear function of the expected indemnity), the premium per unit of insurance coverage will increase with the probability of the loss state. Thus, an effect that is very closely related to the slope of the demand curve for insurance (i.e. the “ordinarity” of insurance) appears when the probability of loss is increased. If insurance is ordinary, then the overall effect of an increase in the probability of loss becomes indeterminate, since insurance will increase as the insured attempts to compensate for the increase in risk, but then it will decrease as the price of insurance is increased.

The case of how the demand for insurance is affected by an increase in the probabiliy of loss when the premium also increases was studied by Jang and Hadar (1995) – here-in after JH – in a model that is very closely related to the standard or traditional model of Mossin. JH consider the effect of an increase in the probability of loss in a two state model, retaining the assumption that the amount of insurable risk is exogenous. JH show that when risk is exogenous, the effect of an increase in the probability of loss is indeterminate if the utility function displays DARA, and that the demand for insurance decreases if the utility function displays either CARA or IARA.

However, as is pointed out by Meyer and Meyer (2004), modelling the demand for insurance holding the insurable risk itself constant is somewhat like modelling the demand for hotdogs while holding the amount of hotdog buns constant. Suffice it to say that when the comparative statics of demand for any good with respect to any relevant parameter is considered in any other model of consumer choice, the amount purchased of the rest of the goods is not held constant. Thus it is realistic and reasonable to consider a model in which the amount of insurable risk is endogenously determined along with the insurance product. In such a setting, when one considers the effect of an increase in the probability of loss upon the optimal demand for insurance, aside from the two effects noted in the standard setting (with an exogenous risk), a third effect must be taken into account – the effect that the increase in the probability of the loss state will have upon the amount of risky asset that is purchased. Clearly, the amount of insurance that is purchased is not independent of the amount of insurable risky asset that is purchased.

Several papers have considered the comparative statics of the demand for insurance when the amount of risky assets is chosen simultaneously with the insurance coverage of them. The most important of these models are:

1. Meyer and Ormiston (1995), here-in after MO.
2. Eeckhoudt, Meyer and Ormiston (1997), here-in after EMO, and
3. Meyer and Meyer (2004), here-in after MM.

All of these models consider settings with a very general description of risk, normally expressed as a continuous state environment. Such a setting, although clearly the most general, has the inconvenience of leading to rather complex and often ambiguous comparative statics. Also, while these three models give a fairly good account of the normality and ordinarity of insurance, and to the case of an increase in the probability of loss conditional upon the premium remaining constant, none of them pays any attention at all to the effect of an increase in the probability of loss for the realistic case in which the premium increases as a result of the greater expected indemnity per unit of coverage.

Concretely, from this literature we have the following relevant results:

Increase in insurance premium

1. MO; insurance is ordinary if DARA and IRRA.
2. EMO; insurance is ordinary if DARA and relative risk aversion not greater than 1.

3. MM; insurance is ordinary if relative risk aversion not greater than 1.

First order stochastic dominant increase in size of loss (premium constant)

1. MO; insurance increases if relative risk aversion not greater than 1.

2. EMO; either insurance increases, or risky asset decreases, or both, if DARA and IRRA and relative risk aversion not greater than 1.

3. MM; insurance increases if relative risk aversion not greater than 1.

More recently, Watt and Loubergé (2004) – here-in after WL – have attempted to simplify the comparative statics of the endogenous risk model by studying the two state setting. Concretely, WL prove the following:

1. When the premium is held constant, an increase in the probability of loss will increase the amount of insurance and decrease the amount of risk purchased, independently of the particular risk aversion characteristic of the individual.

2. If the loss state implies full loss, and if the premium is increased when the probability of the loss state increases, then the optimal purchase of risky assets increases if relative risk aversion is not less than 1. The effect on the optimal purchase of insurance depends upon the value of relative risk aversion, and on the exact characteristics of the relationship between the premium and the probability of loss.

3. If the loss state implies full loss, if the premium is increased when the probability of the loss state increases, and if relative risk aversion is constant and equal to 1 (i.e. logarithmic utility), then the optimal purchase of insurance will increase (resp. decrease, not change) as the marginal effect of probability of loss on the premium is less than (resp. greater than, equal to) the average effect of probability of loss on the premium.

4. If the loss state implies full loss, and if insurance is priced according to a loading factor (i.e. the premium is equal to a loading factor multiplied by the expected indemnity), then an increase in the probability of loss will increase (resp. decrease, not affect) the optimal

purchase of insurance if relative risk aversion is uniformly less than (resp. greater than, equal to) 1.

In WL, in order to arrive at a useful and insightful result on the effect of an increase in the probability of loss on the optimal insurance decision, two simplfying assumptions were made. Firstly, it was assumed that the insurance product is priced according to a standard loading factor equation. This is a very common assumption throughout the literature on the demand for insurance, and so will be retained here. However, the second assumption in WL, that the loss state implies complete loss of the holding of risky asset, is more worrying. In particular, this assumption makes the effects easier to treat because (as is also proven in WL) it provides an absolute guarantee that insurance is both a normal and an ordinary good. It also provides for very simple equations for the effects of increases in risk-free endowed income, and the premium, on the demand for insurance. This contributes to the fact that the effect of an increase in the probability of loss can be stated in a simple and insightful manner, as a dependence upon the value of relative risk aversion.

However, intuition suggests that what is true for the full loss scenario, is likely to be true also when the loss is almost full. That is, if we assume for now that relative risk aversion is greater than 1, then if the loss is full in the loss state, we know that an increase in the probability of the loss state will decrease the optimal purchase of insurance (that is, the negative effect of the increase in the premium outweighs the positive effect of the desire to increase coverage of the increased risk). Then a simple continuity arguement suggests that if the loss state were to imply an almost complete loss, then the same result would still hold. The question then would be if we can find a lower bound on the proportion of the risky asset that is lost in the loss state, and still get the same result as in WL. Even more interestingly, we wonder if a more general statement can be made concerning the relevant effect, depending upon such things as the proportion of the risky asset that is at risk, and the insured's risk aversion characteristics. What is clear is that even though the setting assumed is rather simple (only two states) the equations that are implied by the increase in the probability of the loss state are complex.

2 The model

The problem of an increase in the probability of loss can be very easily seen by the following short analysis that relys on a two state setting (a no-loss state, and a loss state). Denote the optimal purchase of insurance by c^*, the premium per unit of coverage by π, and the probability of the loss state by p. We assume that the premium is a function of the probability of the loss state, $\pi = \pi(p)$, with $\pi'(p) > 0$, that is, an increase in the probability of the loss state leads to an increase in the premium per unit of coverage. Then the effect of an increase in p on c^* is:

$$\frac{\partial c^*}{\partial p} = \left.\frac{\partial c^*}{\partial p}\right|_{\pi} + \frac{\partial c^*}{\partial \pi}\pi'(p)$$

Since we are assuming that $\pi'(p) > 0$, the second summand is negative if insurance is ordinary ($\frac{\partial c^*}{\partial \pi} < 0$). However, the first summand will in general be positive – an increase in the risk implied by an increase in p is countered, at least in part, by an increase in the purchase of insurance. Thus the overall effect is ambiguous, at least until we condition such aspects as the risk aversion characteristic of the individual. Of course, the overall effect is not ambiguous if insurance is not ordinary, that is if $\frac{\partial c^*}{\partial \pi} \geq 0$, since then both terms will be positive and we would have the result that an increase in the probability of the loss state increases the demand for insurance. Naturally, however, that insurance is not ordinary is unrealistic (at least for the case that we are interested in here of endogenous risk).

Thus the sign of the overall effect depends on the relative strengths of the risk combating term $(\left.\frac{\partial c^*}{\partial p}\right|_{\pi})$ and the price effect $(\frac{\partial c^*}{\partial \pi}\pi'(p))$. Naturally, these are not in general simple expressions. For example, the first part of the price effect, $\frac{\partial c^*}{\partial \pi}$, is the slope of the demand curve for insurance, which will itself be comprised of an income and a substitution effect.

The problem of the individual is to maximise her expected utility from the purchase of a risky asset, x, at a unit price v, and an insurance product, c, at a unit price π, subject to a budget constraint and subject to the individual not being allowed to sell insurance. We assume that in the loss state, which occurs with probability p, a proportion a of the risky portfolio is lost, but the insurance product then pays an indemnity of c. We do not allow for a risk-free asset, as doing so implies that the demand for either the insurance product or for the risky asset is zero. It turns out that this is equivalent to assuming that the price of a unit of risk free terminal wealth purchased as the risk free asset, which is assumed to be 1, is not less than the price of a fully insured unit of risky asset; that is $v + a\pi \leq 1$ (see WL for more details). Thus, the problem is:

$$\max_{x,c} U(x,c) \equiv (1-p)u(x) + pu((1-a)x + c)$$

$$\text{subject to } vx + \pi c \leq w \text{ and } c \leq ax$$

The individual's initital wealth, w, is assumed to include all borrowing capacity, which in turn is assumed to be strictly finite. We will indicate the solution to this problem by the vector (x^*, c^*).

Since the objective function is concave, and the restrictions are linear, we know that there exists a unique optimum for the problem. The Lagrangian is:

$$L(x,y,\delta) = (1-p)u(x) + pu((1-a)x + c) + \delta_1(w - vx - \pi c) + \delta_2(ax - c)$$

where δ_1 is the multiplier corresponding to the budget constraint, and δ_2 is the multiplier corresponding to the coverage restriction (naturally, we restrict $\delta_i \geq 0$ for $i = 1, 2$). The first order conditions for the optimum are

$$\frac{\partial L}{\partial x} = (1-p)u'(x^*) + pu'((1-a)x^* + c^*)(1-a) - v\delta_1 + a\delta_2 = 0 \tag{1}$$

$$\frac{\partial L}{\partial c} = pu'((1-a)x^* + c^*) - \pi\delta_1 - \delta_2 = 0 \tag{2}$$

and the complementary slackness conditions are

$$\delta_1(w - vx^* - \pi c^*) = 0 \text{ and } \delta_2(ax^* - c^*) = 0 \tag{3}$$

Firstly, note that from (1) it must be true that $\delta_1 > 0$, so the budget constraint must always saturate:

$$w = vx^* + \pi c^* \tag{4}$$

Secondly, assuming an internal solution ($\delta_2 = 0$), from (1) and (2) we get the tangency condition in state contingent space:[1]

$$\frac{(1-p)u'(x^*) + pu'((1-a)x^* + c^*)(1-a)}{pu'((1-a)x^* + c^*)} = \frac{v}{\pi}$$

[1] Of course, this is nothing more than the usual condition of equality between the marginal rate of substitution and the ratio of state-contingent prices.

Simple operations on this equation lead directly to

$$\frac{u'(x^*)}{u'((1-a)x^*+c^*)} = \frac{p(v-\pi(1-a))}{\pi(1-p)} \tag{5}$$

Clearly, an internal solution can only occur if $v - \pi(1-a) > 0$, which will be assumed from now on.[2]

In order to continue, we now make an important assumption:

ASSUMPTION 1: We assume from now on that insurance is priced according to a loading factor; $\pi(p) = kp$, with $k \geq 1$.

Recall that above we assumed that no risk-free good exists, which is equivalent to assuming that $v + a\pi \leq 1$. Under assumption 1 this now reads $v + kpa \leq 1$, or $v \leq 1 - kpa$. However, WL's proposition 1 shows that in this model there will always be an internal solution, $c^* < ax^*$, when $\pi > \frac{pv}{1-pa}$. Thus under assumption 1, the solution will be internal if $kp > \frac{pv}{1-pa}$, that is if $k > \frac{v}{1-pa}$. Clearly, this is equvalent to $v < k - kpa$. However, since $k \geq 1$, it holds that $1 - kpa \leq k - kpa$. And so since we are assuming that $v \leq 1 - kpa$ then we are also assuming an internal solution to the problem. Thus from now on we shall use the inequality $c^* < ax^*$ whenever appropriate.

Our objective is to study how the two equations defining the optimal solution, (4) and (5), are affected by an increase in p. Under assumption 1, these two equations can be simplified by eliminating the unit price of insurance. The budget constraint becomes

$$w = vx^* + kpc^* \tag{6}$$

and the tangency condition becomes

$$\frac{u'(x^*)}{u'((1-a)x^*+c^*)} = \frac{(v-kp(1-a))}{k(1-p)} \tag{7}$$

The effect of an increase in p on (6) is:

$$0 = v\frac{\partial x^*}{\partial p} + kc^* + kp\frac{\partial c^*}{\partial p} \tag{8}$$

[2] Recall that, we are already assuming $v + a\pi \leq 1$, and now we also have $v + a\pi > \pi$, so in short, our assumption is $\pi < v + a\pi \leq 1$. This new assumption only implies that some insurance will always be purchased.

And the effect of an increase in p on (7) is (after simplification on the right-hand-side):

$$\frac{u''(x^*)\frac{\partial x^*}{\partial p}u'((1-a)x^*+c^*)) - u'(x^*)u''((1-a)x^*+c^*)\left((1-a)\frac{\partial x^*}{\partial p}+\frac{\partial c^*}{\partial p}\right)}{u'((1-a)x^*+c^*)^2}$$

$$= \frac{v-k(1-a)}{k(1-p)^2} \tag{9}$$

Note that (8) and (9) are two simultaneous equations in the unknowns $\frac{\partial x^*}{\partial p}$ and $\frac{\partial c^*}{\partial p}$. These are the two equations that we must use in order to sign these two unknowns.

However, (9) can be simplified using (5) and the Arrow-Pratt measure of absolute risk aversion, $R_a(z)$. It is not difficult to show that the effect can be expressed as:

$$R_a((1-a)x^*+c^*)\left((1-a)\frac{\partial x^*}{\partial p}+\frac{\partial c^*}{\partial p}\right) - R_a(x^*)\frac{\partial x^*}{\partial p} = \frac{v-k(1-a)}{(v-kp(1-a))(1-p)}$$

that is

$$\frac{\partial x^*}{\partial p}\left((1-a)R_a((1-a)x^*+c^*) - R_a(x^*)\right) + \frac{\partial c^*}{\partial p}R_a((1-a)x^*+c^*) =$$

$$= \frac{v-k(1-a)}{(v-kp(1-a))(1-p)} \tag{10}$$

One straightforward result is immediately evident. From (8) it is clear that both $\frac{\partial x^*}{\partial p}$ and $\frac{\partial c^*}{\partial p}$ cannot be simultaneously positive. Thus they are either both negative or one is positive and the other is negative. In order to say more, it is convenient to solve the two equations in their unknowns. Since we are only interested in studying $\frac{\partial c^*}{\partial p}$, we shall concentrate on that unknown.

Solving (8) for $\frac{\partial x^*}{\partial p}$, substituting into (10), and simplifying yeilds the following result

$$\frac{\partial c^*}{\partial p}\left[R_a(y^*)(v-k(1-a)) + R_a(x^*)kp\right] = \frac{v(v-k(1-a))}{(1-p)(v-kp(1-a))} + kc^*\left[(1-a)R_a(y^*) - R_a(x^*)\right] \tag{11}$$

where $y \equiv (1-a)x + c$. Since $R_a(y^*)(v-k(1-a)) + R_a(x^*)kp > 0$, the sign of $\frac{\partial c^*}{\partial p}$ is the same as the sign of the entire right-hand-side of (11). The second term on the right-hand-side is positive (negative) whenever insurance is inferior (normal) (see WL for a proof), but it is the first term on the right-hand-side that is more complicated. Since we have made the assumption that $v - kp(1-a) > 0$, the denominator of the first term on the right-hand-side is positive, however, the numerator may be either positive or negative, or equal to zero. In any case, using (11) we can make some further statements that require no proof:

Proposition 1 *If insurance is normal and if $v \leq k(1-a)$, then $\frac{\partial c^*}{\partial p} < 0$. If insurance is inferior and if $v \geq k(1-a)$, then $\frac{\partial c^*}{\partial p} > 0$.*

However, as we have just noted, in general we have

$$\text{sign}\frac{\partial c^*}{\partial p} = \text{sign}\left[\frac{v(v-k(1-a))}{(1-p)(v-kp(1-a))} + kc^*\left[(1-a)R_a(y^*) - R_a(x^*)\right]\right]$$

that is

$$\frac{\partial c^*}{\partial p} \gtreqless 0 \ \Leftrightarrow\ \frac{v(v-k(1-a))}{(1-p)(v-kp(1-a))k} \gtreqless -c^*\left[(1-a)R_a(y^*) - R_a(x^*)\right] \tag{12}$$

If we define

$$\begin{aligned} \frac{v(v-k(1-a))}{(1-p)(v-kp(1-a))k} &\equiv g(a) \\ -c^*\left[(1-a)R_a(y^*) - R_a(x^*)\right] &\equiv h(a) \end{aligned}$$

then we have

$$\frac{\partial c^*}{\partial p} \gtreqless 0 \ \Leftrightarrow\ g(a) \gtreqless h(a)$$

Let us consider the two functions $g(a)$ and $h(a)$ in more detail. Let us begin with $g(a)$, which is the least complex of the two. We have:

1. $g(1) = \frac{v}{k(1-p)}$. However, recall that we are assuming an internal solution at all values of a, which requires that $\pi > \frac{pv}{1-pa}$. With $\pi = kp$ this reads $1 > \frac{v}{k(1-pa)}$, and so at $a = 1$ we have $g(1) < 1$.

2. $g'(a) = \left(\frac{v}{v-kp(1-a)}\right)^2$. Thus, clearly we have $g'(a) > 1$ at all $a < 1$ and $g'(a) = 1$ at $a = 1$. Therefore, the graph of $g(a)$ is an increasing function, with slope greater than 1 at all internal points, that cuts the a axis at the point $\frac{k-v}{k}$, and that reaches height $\frac{v}{k(1-p)}$ at $a = 1$. It is also straightforward to show that the second deriviative of $g(a)$ is negative, so that it is a strictly concave function. Such a graph is shown in figure 1.

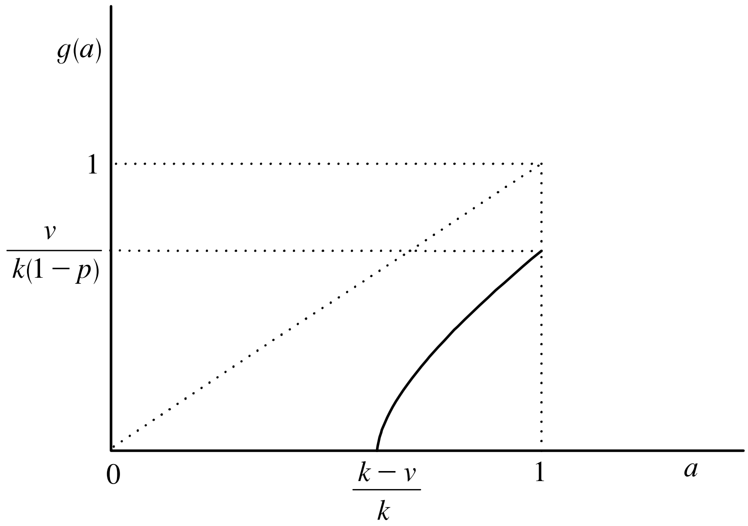

Figure 1:

Now, consider the function $h(a)$. We have

1. $h(1) = c^* R_a(x^*)$. Even though we are assuming relative risk aversion greater than 1, since $c^* < x^*$, clearly $h(1) < R_r(x^*)$, and so $h(1)$ may be greater than, less than, or equal to 1. However, in WL it is proven that if the Arrow-Pratt measure of relative risk aversion, $R_r(z)$, is greater than 1 then we get $\frac{\partial c^*}{\partial p} < 0$ at $a = 1$. Thus, assuming $R_r(z) > 1$, then we know that $h(1) > g(1)$. Secondly, it is also true that $h(0) = c^*(0)\left[R_a(y^*(0) - R_a(x^*(0)\right]$. However, since $y = (1-a)x + c$, we have $y^*(0) = x^*(0) + c^*(0)$. But the problem is conditioned such that $0 \le c \le ax$, and at any internal solution $y^* \le x^*$. Thus clearly, when $a = 0$, that is, no loss can be had, we must have $c^*(0) = 0$, and so $y^*(0) = x^*(0)$. Thus $h(0) = 0$, that is, the graph of $h(a)$ passes through the origin.

2. On the other hand, the slope of $h(a)$ can be seen to be

$$h'(a) = -\frac{\partial c^*}{\partial a}\left[(1-a)R_a(y^*) - R_a(x^*)\right] - c^*\left[-R_a(y^*) + (1-a)R_a'(y^*)\frac{\partial y^*}{\partial a} - R_a'(x^*)\frac{\partial x^*}{\partial a}\right] \tag{13}$$

At this point, we recall a useful result that is proven in WL.

Proposition 2 *$\frac{\partial c^*}{\partial a} > 0 > \frac{\partial x^*}{\partial a}$, and $\frac{\partial y^*}{\partial a} = -\frac{\partial x^*}{\partial a} > 0$.*

Using this, we can write (13) as:

$$h'(a) = -\frac{\partial c^*}{\partial a}\left[(1-a)R_a(y^*) - R_a(x^*)\right] + c^* R_a(y^*) + c^* \frac{\partial x^*}{\partial a}((1-a)R'_a(y^*) + R'_a(x^*)) \qquad (14)$$

Assuming that insurance is not inferior, then $(1-a)R_a(y^*) - R_a(x^*) < 0$ (see WL for a proof), and since from Result 2 here $\frac{\partial c^*}{\partial a} > 0$, the first summand of (14) is positive. The second summand is positive under positive absolute risk aversion, and assuming decreasing absolute risk aversion, and recalling that (see Result 2) $\frac{\partial x^*}{\partial a} < 0$, the third term is also positive. Therefore, under the realistic assumption of DARA we have $h'(a) > 0$ for all $a > 0$. At $a = 0$, since (as we have just argued above) $y^*(0) = x^*(0)$, the first summand in (14) becomes 0, and since $c^*(0) = 0$, the second and third summands are also 0, and so it turns out that $h'(0) = 0$. In short, we know that, assuming (i) positive decreasing absolute risk aversion, (ii) relative risk aversion uniformly greater than 1, and (iii) insurance is a normal good, then the function $h(a)$ goes through the origin with exactly zero slope, has positive slope for all $a > 0$, and reaches a value at $a = 1$ that is greater than $\frac{v}{k(1-p)}$. Such a function is shown in figure 2.

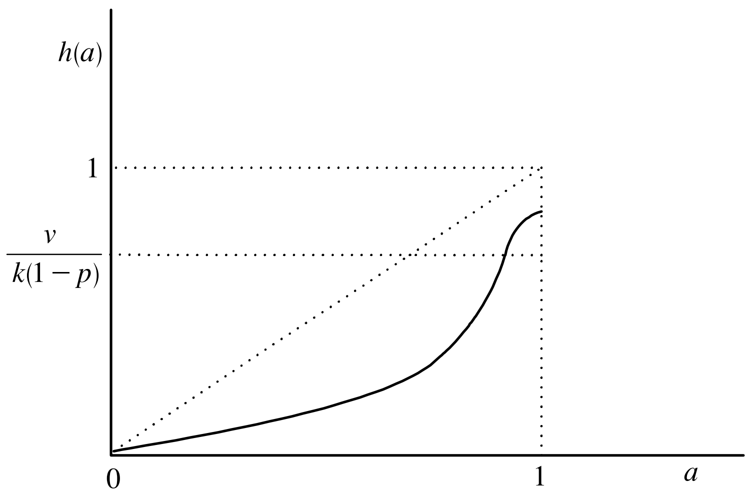

Figure 2:

What is clear from figures 1 and 2 is that, under the assumptions required to generate these two functional forms, we have

Proposition 3 *Assuming that insurance is a normal good,* $h(a) > g(a)$ *for sure if* a *is either sufficiently large, or sufficiently small. Concretely, for all* $a \leq \frac{k-v}{k}$*, this is clearly the case (since* *in this case $g(a) < 0 < h(a)$). In other words, it can only hold that $\frac{\partial c^*}{\partial p} > 0$ for intermediate values of a, although that such a case actually exists cannot in general be guaranteed.*

The fact that when $a \leq \frac{k-v}{k}$ mimics the result already noted in Proposition 2 for the case of insurance being a normal good, since $a \leq \frac{k-v}{k}$ reorders to $v \leq k(1-a)$. In any case, Proposition 4 also clearly rules out the possibility that $\frac{\partial c^*}{\partial p} > 0$ for all a. Thus, it provides limited support for an increase in the probability of loss having the effect of decreasing the demand for insurance.

Finding a condition for $\frac{\partial c^*}{\partial p} < 0$ for all a is far more challenging. Such would be the case, for example, if it were to hold that $h'(a) < g'(a)$ for all a, since in that case it becomes impossible for the function $h(a)$ to touch the function $g(a)$, and so the former is always higher than the latter (recall that $h(1) > g(1)$ under our assumption on risk aversion). Also, we could have $h'(a) < 1$ at all points, in which case the function $h(a)$ would lie everywhere above the diagonal of the graph. This would be the case, for example, if insurance were neither inferior nor normal (first term of (14) is zero), if absolute risk aversion were constant (third term of (14) is zero), and if $c^* R_a(y^*) < 1$.

3 Conclusions

In this paper we have reconsidered the effect of an increase in the probability of loss upon the demand for insurance. In the existing literature, this effect has been shown to be ambiguous when the increase in probability is reflected in an increased premium, and when the insured risk is exogenous. Here, we extend the existing models by allowing the insured risk to be endogenous. In this setting, the effect is still ambiguous, although we are able to make some conditional statements. However, the paper provides a certain degree of support for the resulting effect of an increase in the probability of loss to be a reduction in insurance.

Firstly, as has been shown in the past literature, it is reasonably likely that insurance is a normal good in the sence of traditional microeconomic theory. In that case, we have shown that if $v \leq k(1-a)$ then indeed an increase in the probability of loss will decrease the demand for insurance. Clearly, this condition is easier to satisfy the smaller is the proportion of the risky asset that is at risk (i.e. the smaller is a), the greater is the loading factor k, and the smaller is the unit price of the risky asset v. Thus an increase in the probability of loss will decrease the demand for insurance when the proportion of the risky asset that may be lost is sufficiently small. However, we have also shown that the effect is the same when the proportion of the risky asset that may be lost is sufficiently large. Thus, if an increase in the probability of loss does ever have the effect of increasing the demand for insurance then the proportion of the risky asset that is at risk cannot be either too high or too low.

References

Eeckhoudt, L., J. Meyer and M. Ormiston (1997), "The Interaction Between the Demands for Insurance and Insurable Assets", *Journal of Risk and Uncertainty*, **14**, 25-39.

Jang, Y. and J. Hadar (1995), "A Note on Increased Probability of Loss and the Demand for Insurance", *The Geneva Papers on Risk and Insurance Theory*, **20**, 213-216.

Meyer, D., and J. Meyer (2004), "A More Reasonable Model of Insurance Demand", in Aliprantis, C.D, K.J. Arrow, P. Hammond, F. Kubler, H.M. Wu, and N.C. Yannelis (Eds.), *Assets, Beliefs, and Equilibria in Economic Dynamics. Essays in Honor of Mordecai Kurz*, Studies in Economic Theory (vol. 18), Springer-Verlag, pp. 733-742.

Meyer, J., and M. Ormiston (1995), "Demand for Insurance in a Portfolio Setting", *The Geneva Papers on Risk and Insurance Theory*, **20**, 203-211.

Mossin, J. (1968), "Aspects of Rational Insurance Purchasing", *Journal of Political Economy*, **76**, 553-568.

Watt, R. and H. Loubergé (2004), "On the Demand for Budget Constrained Insurance", paper presented to the EGRIE Annual Congress, Marseille, France, September 2004. Available on http://papers.ssrn.com/sol3/papers.cfm?abstract_id=691101