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The Geneva Association - General Secretariat - 53, route de Malagnou - CH-1208 Geneva
Tel.: +41-22-707 66 00 - Fax: +41-22-736 75 36 - secretariat@genevaassociation.org - www.genevaassociation.org

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On the Possibility of Profitable Self-Selection Contracts in Competitive Insurance Markets

Arthur Snow

Department of Economics
Terry College of Business
University of Georgia
Athens, GA 30602

Abstract

Several studies have extended the model of competitive insurance contracting with adverse selection introduced by Rothschild and Stiglitz (1976) to incorporate additional dimensions of private information, and have concluded that (jointly) profitable contracts may survive in a separating, self-selection equilibrium. In each such instance, one contract breaks even, while the other contract not only earns a positive profit, but any alternative contract with a lower premium and comparable coverage would attract all applicants and earn negative profit. However, these profitable self-selection contracts cannot survive in a competitive equilibrium where insurers can enter and exit freely and costlessly offer as many different types of contracts as they wish. Under the competitive conditions of the Rothschild-Stiglitz environment, insurers can offer one contract that earns negative profit provided these losses are offset by positive profit earned on another contract. There always exists such a pair of defections from the profitable self-selection contracts proposed as equilibria for the modified Rothschild-Stiglitz environments, implying that these contracts cannot be sustained as Nash equilibria. In these instances, Nash equilibrium is the breakeven pooling contract identified as the strategically stable outcome of the three-stage representation of insurance contracting proposed by Hellwig (1987).

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1. Introduction

Several studies have extended the model of competitive insurance contracting introduced by Rothschild and Stiglitz (1976), in which otherwise identical insurance applicants differ with respect to their privately known loss probabilities, by incorporating additional dimensions of private information. Smart (1996) and Villeneuve (2003) assume that applicants also differ in their privately known degrees of risk aversion, Wambach (2000) introduces heterogeneity with respect to privately known initial wealth, and Sonnenholzner and Wambach (2004) analyze a two-period model in which applicants differ with respect to their privately known rates of time preference and can privately invest in self-protection, adding an element of moral hazard. In each of these studies, there arises the possibility that the separating, breakeven contracts identified by Rothschild and Stiglitz as the only candidate for a pure-strategy Nash equilibrium are not incentive compatible in the modified contracting environment. In these instances, an alternative separating pair is identified as the competitive equilibrium, with one of the contracts breaking even while the other earns positive profit.

While each of these studies enriches the stylized Rothschild-Stiglitz model by adding realistic features of insurance contracting environments, the survival of (jointly) profitable contracts in equilibrium is inconsistent with the possibility of free entry and exit by firms and their contractual offerings, which are distinguishing characteristics of competitive markets. Indeed, the principal result established in this paper is that the presence of (jointly) profitable contracts in the proposed equilibria indicates that there is no pure strategy Nash equilibrium for these modified Rothschild-Stiglitz environments in the two-stage representation of competitive contracting in which insurers make irrevocable contact offers to applicants in the first stage and applicants choose their preferred contract in the second. The nonexistence of equilibrium in the two-stage game is established in section3 and resolved by appealing to the three-stage screening game introduced by Hellwig (1987).

Profitable contracting does not always arise in the modified Rothschild-Stiglitz environments, but when it does the high-risk contract breaks even and the incentive constraint for high-risk applicants is binding, while the low-risk contract earns a positive (expected) profit.[1] However, this configuration cannot be a Nash equilibrium, since a defecting insurer can offer a pair of self-selection contracts attractive to both risk classes, an unprofitable contract with full coverage that attracts only high-risk applicants and a profitable contract attractive to low risks, with the pair of contracts being jointly profitable. As long as there is free entry and exit for insurers, these defections are entered on the market and the proposed configurations with profitable self-selection contracts fail to survive as equilibria.

Rothschild and Stiglitz demonstrated that the two-stage representation of insurance contracting does not have a pure strategy Nash equilibrium if there are too few high risks in the applicant pool. Hellwig (1987) resolved this nonexistence problem by proposing a three-stage representation of the contracting environment wherein the uniformed insurers move first, as in the two-stage representation proposed by Rothschild and Stiglitz, but can choose in the third stage to renege on their initial offers. Hellwig found that a unique, zero-profit, strategically stable, pure strategy Nash equilibrium always exists and corresponds to the Rothschild-Stiglitz pair when that pair can be sustained as a Nash equilibrium in the two-stage game, and otherwise corresponds to the breakeven pooling contract most preferred by low-risk applicants.

In the modified Rothschild-Stiglitz environments studied by Smart (1996), Villeneuve (2003), Wambach (2000), and Sonnenholzner and Wambach (2004), the Rothschild-Stiglitz self-selection contracts are not sustainable as a Nash equilibrium, regardless of the proportion of high risks in the applicant pool, when low risks can be attracted to an incentive compatible and profitable contract. In these instances, the only candidate for Nash equilibrium in the two-stage game is a profitable self-selection pair. However, no such pair can be sustained as a Nash equilibrium under competitive conditions of free entry and exit with respect to insurers and their contractual offers. This nonexistence problem can be resolved by recognizing that insurers may renege on their contractual offers in a third stage of the contracting game, as suggested by Hellwig. The strategically stable Nash equilibrium for the three-stage screening game is the breakeven pooling contract most preferred by low risks.

The model introduced by Rothschild and Stiglitz and the nonexistence problem they identified are reviewed in the next section. The possibility of profitable equilibrium contracts in the modified Rothschild-Stiglitz environments analyzed by Smart (1996) and Villeneuve (2003) is examined in section 3. The nonexistence of equilibrium in these models is established and its resolution is discussed with reference to alternative Nash equilibrium concepts.[2] The related model analyzed by Wambach (2000) and the more complex model examined by Sonnenholzner and Wambach (2004) are discussed in section 4. Conclusions are presented in section 5.

2. Equilibrium when Risk Type is Hidden Knowledge

Rothschild and Stiglitz (1976) posit an environment with two large classes of individuals, each of whom possesses the von Neumann-Morgenstern utility function U and initial wealth W. Each individual faces an independent risk of suffering a loss D, and individuals differ only in that those in one class incur the loss with a high probability p^H, while those in the other class incur the loss with a low probability $p^L < p^H$. The probability of loss is hidden knowledge. It is assumed that insurers have costless access to the information and legal instruments needed to exercise exclusivity, and that, in equilibrium, each insurer attracts a representative sample of the pool of applicants so that each contract offered earns its expected profit with probability one. Except for the hidden knowledge of loss probabilities, the environment is perfectly competitive and would provide all applicants with full and fair insurance if loss probabilities were common knowledge.

Rothschild and Stiglitz assume that each insurer offers an applicant the opportunity to choose from a menu of contracts that specify a nonnegative premium and a nonnegative deductible. Each insurance applicant, in turn, selects a most preferred contract from among those offered on the market. For this two-stage game, a Nash equilibrium in pure strategies is a pair of contracts such that, when individuals select their most preferred contracts, each contract earns a nonnegative profit, and there is no other pair of contracts that would attract some individuals away from the equilibrium pair and earn positive profit.

Rothschild and Stiglitz established that the only candidate for equilibrium is the separating pair that provides full and fair insurance to high-risk applicants and fair insurance to low risks, with high risks being indifferent between their contract and the contract chosen by low risks. This pair will be referred to as the Rothschild-Stiglitz contracts. Rothschild and Stiglitz also showed that these contracts cannot be sustained as an equilibrium if the proportion of high risks in the population of applicants is sufficiently low that both risk types prefer a profitable pooling contract over their Rothschild-Stiglitz contracts. It is now well known that the defection need not be a profitable pooling contract.[3] It suffices that a pair of incentive compatible contracts exists such that the members of each risk class prefer their defecting contract to their Rothschild-Stiglitz contract, while the two defecting contracts earn jointly nonnegative profit, with positive profit earned on the low-risk contract offsetting losses incurred on the high-risk contract.

The contractual pairs that bound the set of potential defections from the Rothschild-Stiglitz contracts can be identified by characterizing those pairs that jointly break even, provide full coverage for high risks, and satisfy the self-selection constraint for high risks with equality while attracting them away from their full and fair insurance contract. Let W^H denote a sure level of wealth for high-risk applicants, and let (W_1^L, W_2^L) denote a partially insured, state-contingent wealth for low-risk applicants, with state 2 being the loss state. The contracts underlying W^H and (W_1^L, W_2^L) jointly break even if expected wealth per applicant is equal to the expected per-applicant wealth endowment, so that

$$\lambda W^H + (1-\lambda)[(1-p^L)W_1^L + p^L W_2^L] = W - \bar{p}D, \qquad (1)$$

where λ denotes the proportion of high risks in the applicant pool and $\overline{p} \equiv \lambda p^H + (1-\lambda) p^L$ denotes the applicants' average loss probability. The underlying contracts are incentive compatible if they satisfy the high-risk self-selection constraint with equality, so that

$$U(W^H) = (1-p^H)U(W_1^L) + p^H U(W_2^L)\,. \tag{2}$$

Finally, the high-risk contract underlying W^H is attractive to these applicants if

$$W^H \in (W - p^H D, W - \overline{p}D]\,, \tag{3}$$

where the lower bound is the wealth guaranteed by the high-risk Rothschild-Stiglitz contract, and the upper bound ensures that the low-risk contract does not lead to over-insurance.

In Figure 1, the locus *FA* represents the low-risk contingent wealth positions (W_1^L, W_2^L) that satisfy conditions (1)-(3). Point *E* represents the contingent wealth endowment $(W, W-D)$, and the line segments H^*E and L^*E represent the fair-odds lines for high-risk and low-risk applicants, respectively, along which their contracts break even. Point *F* corresponds to the first-best pooling contract, and the line segment *FE* represents the fair-odds line for pooling contracts.[4] Points H^* and A correspond to the high-risk and low-risk Rothschild-Stiglitz contracts, respectively. The Figure illustrates a case in which the locus *FA* rises above the indifference curve for low-risks associated with their Rothschild-Stiglitz contract, in which event there is no pure strategy Nash equilibrium for the two-stage representation of competitive insurance contracting. Notice that conditions (1)-(3), which determine the locus *FA*, are independent of the preferences of low-risk applicants.

There is a critical value for the population proportion of high risks, λ^*, such that the Rothschild-Stiglitz contracts constitute a Nash equilibrium in the two-stage game if and only if λ is greater than or equal to λ^*.[5] For environments in which λ is less than λ^*, as in Figure 1, there is no pure strategy Nash equilibrium for the two-stage screening game. By contrast, in three-stage games of contracting with adverse selection, there are two candidates for Nash equilibrium: the separating pair of Rothschild-Stiglitz contracts and the breakeven pooling contract most preferred by low risks.

Cho and Kreps (1987) analyzed a signaling version of the game in which the informed parties move first and can renege on their offers in the third stage, and demonstrated that the strategically stable Nash equilibrium is the separating pair of Rothschild-Stiglitz contracts for all values of $\lambda \in (0,1)$. Hellwig (1987) proposed a screening version of the game in which the uniformed parties move first and can then renege in the third stage, and found that the strategically stable Nash equilibrium is the Rothschild-Stiglitz separating pair of contracts when they can be sustained as a Nash equilibrium in the two-stage game $(\lambda \geq \lambda^*)$, but when they cannot be so sustained $(\lambda < \lambda^*)$ the equilibrium is the breakeven pooling contract most preferred by low risks. Accepting that the screening model, rather than the signaling model, offers the more realistic representation of contracting in insurance markets, pooling is the predicted response to the nonexistence problem posed by Rothschild and Stiglitz.

Finally, since insurance applicants differ only with respect to their loss probabilities, their indifference curves have the Spence-Mirrlees single-crossing property. One consequence is that there are profitable cream-skimming defections from any fair pooling contract, ensuring that pooling cannot be a Nash equilibrium in the two-stage game. A second consequence is that the low-risk Rothschild-Stiglitz contract is their most preferred contract among those that earn a nonnegative profit while satisfying the high-risk self-selection constraint. This may no longer be the case when indifference curves do not have the single-crossing property.

3. Equilibrium when Risk Type and Risk Aversion Are Hidden Knowledge

Smart (1996) and Villeneuve (2003) extend the Rothschild-Stiglitz model by allowing for individuals to differ not only with respect to their probabilities of suffering a loss, but also with respect to their degrees of risk aversion.[6] In these modified Rothschild-Stiglitz environments, risk aversion is hidden knowledge along with risk of loss. Both studies argue that profitable equilibria can arise in these environments when the Spence-Mirrlees single-crossing property is violated. The simple case of homogeneous risk classes will be examined first, followed by an analysis of heterogeneous risk classes. In each case, it will be established that the proposed profitable equilibria cannot be Nash equilibria in a competitive environment of free entry and exit. Like the Rothschild-Stiglitz nonexistence problem, this one can be resolved by appealing to the strategically stable equilibrium of the three-stage screening game.

A. Homogeneous Risk Classes

In the simplest case that gives rise to the possibility of positive profits, all members of each risk class are equally risk averse, but low risks are more risk averse than high risks. In the relevant case, low-risk applicants are so risk averse that the Spence-Mirrlees single-crossing property is violated, and the indifference curves of low risks cross the indifference curves of high risks twice. As a consequence of this double crossing, it may be possible to attract low risks away from their Rothschild-Stiglitz contract by offering an alternative that satisfies the high-risk self-selection constraint and earns a positive profit.

A.1. Nonexistence of Equilibrium in Two-Stage Games

Figure 2 illustrates this possibility. Point *B* results from the contract most preferred by low risks among those that satisfy the high-risk self-selection constraint associated with their Rothschild-Stiglitz contract. While the high-risk contract breaks even, the low risk contract earns a positive profit, and therefore the Rothschild-Stiglitz contracts cannot be sustained as an equilibrium for any value of $\lambda \in (0,1)$ under Nash behavior in any of the proposed games of competitive contracting in the presence of adverse selection.

Recall that the locus *FA*, which represents the low-risk wealth positions for profitable defections from the Rothschild-Stiglitz pair, is independent of their risk preferences. Hence, the assumption that they are more risk averse than high-risk types has no bearing on the position of locus *FA*. Next, observe that, regardless of the value of λ and the position of point *F* along the 45-degree line, the locus *FA* must pass above point *B*.[7] It follows that a pair of jointly profitable contracts exists such that one contract attracts high risks away from H^* to an alternative contract offering them full insurance and the other attracts low risks away from *B* while satisfying the high-risk self-selection constraint. As a consequence, the contracts underlying H^* and *B* cannot be sustained as a Nash equilibrium for any value of λ in the two-stage representation of insurance contracting.

A.2. Nash Equilibrium in Three-Stage Screening Games

Observe that the contracts underlying H^* and *B* are the only candidates for a separating equilibrium, since the double crossing of indifference curves rules out the Rothschild-Stiglitz pair, and cross-subsidized separating contracts cannot survive in equilibrium.[8] Moreover, as in the original Rothschild-Stiglitz model, the breakeven pooling contract most preferred by low risks is the only candidate for a pooling equilibrium. Recognizing that insurers propose offers that they can subsequently withdraw, the three-stage screening game is well suited to resolving the nonexistence of equilibrium in the two-stage game. Since the separating contracts underlying H^* and *B* never constitute an equilibrium in the two-stage game, the predicted outcome is the breakeven pooling contract most preferred by low risks.

A.3. Nash Equilibrium in Games with Imperfect Competition

To sustain as a Nash equilibrium the contracts underlying points H^* and *B* illustrated in Figure 2, Villeneuve (2003) assumes that insurers are restricted to offering only one type of contract.[9] However, the restriction to one type of contractual offering represents a departure from competitive conditions in which insurers are free to offer as many types of contracts as they wish. Smart (2000) takes a different tack and assumes that insurers must overcome a fixed entry barrier, and that entry occurs until rent on the contract underlying *B* is dissipated by the average entry cost per applicant. This too represents a departure from competitive conditions where free entry and exit is the hallmark.[10] Thus, while the contracts underlying H^* and *B* can be sustained under imperfectly competitive conditions, they cannot be sustained under competitive conditions of free entry and exit with respect to insurers and their contractual offers.

B. Heterogeneous Risk Classes

The nonexistence result established for homogeneous risk classes in the preceding section applies equally to environments with heterogeneous risk classes. Whenever the Rothschild-Stiglitz equilibrium cannot be sustained because of double-crossing indifference curves, there is no pure strategy Nash equilibrium in the two-stage game of competitive insurance contracting, and no equilibrium with (jointly) profitable self-selection contracts.

B.1. Nonexistence of Equilibrium in Two-Stage Games

If some members of the high-risk class are as risk averse as the low risks, then the profitable low-risk contract cannot result in a contingent wealth position on a high-risk, low risk-tolerant indifference curve, since these high-risk types have the same von Neumann-Morgenstern utility function as the low risks, and therefore the indifference curves of the two types must have the Spence-Mirrlees single-crossing property. As a consequence, the resulting wealth position, point *B*, must lie along the high-risk, high risk-tolerant indifference curve, which must intersect the low-risk fair-odds lie at the point associated with the low-risk Rothschild-Stiglitz contract, since the intersection of the fair-odds line with the indifference curve of high-risk, low-risk tolerant types would have a lower deductible and would attract the high-risk, high risk-tolerant types and earn negative profit. Therefore, the contractual configuration is again depicted in Figure 2, and the locus *FA* must rise above point *B*, eliminating the contracts underlying H^* and *B* as a Nash equilibrium.

Now suppose that some high risks are more risk averse than others, but not as risk averse as the low risks, and that low-risk indifference curves cross twice the indifference curves of both types of high risks. There now arises the possibility that the profitable low risk contract results in a contingent wealth position that lies along the high-risk, low risk-tolerant indifference curve, as illustrated by point B' in Figure 3. Let θ denote the proportion of high-risks with high risk-tolerance. Point B' attracts these high risks and is preferred to *B* by low risks. Let θ^* denote the critical value for θ such that profits earned on low risks at B' just balance the losses incurred on high risks. Then, when θ equals θ^*, the contracts underlying H^* and B' break even individually and constitute a Nash equilibrium with partial pooling in the two-stage representation of insurance contracting.

When θ exceeds θ^*, the contract underlying B' is unprofitable and the low-risk contract yields point *B* along the indifference curve of the high-risk, high risk-tolerant types. Once again, locus *FA* must rise above point *B* and the contracts underlying H^* and *B* cannot be sustained as a Nash equilibrium. On the other hand, when θ is less than θ^*, the contract underlying B' is again a partial pooling contract, but is now profitable. However, this contract is not sustainable.

To establish that the contract underlying point B' is not sustainable as a profitable partial pooling contract, let (W_1^L, W_2^L) denote the contingent wealth associated with B', and let *U* and *V* be the utility functions for low risks and for high risks with low-risk tolerance, respectively. At B', the indifference curves for the corresponding expected utility functions of these types are tangent, so that

$$\frac{(1-p^L)U'(W_1^L)}{p^L U'(W_2^L)} = \frac{(1-p^H)V'(W_1^L)}{p^H V'(W_2^L)}. \qquad (4)$$

Consider a differential change in the contract that increases the premium while holding constant the expected utility of low risks, so that $dW_1^L < 0$ and

$$dW_2^L = -\frac{(1-p^L)U'(W_1^L)}{p^L U'(W_2^L)} dW_1^L. \tag{5}$$

This change increases the profit earned on low risks, since equation (5) implies

$$-(1-p^L)dW_1^L - p^L dW_2^L = -(1-p^L)\left[1-\frac{U'(W_1^L)}{U'(W_2^L)}\right] dW_1^L > 0, \tag{6}$$

where the inequality follows given $dW_1^L < 0$ and $W_2^L < W_1^L$. Moreover, the tangency condition (4) implies that profit earned on the high-risk, high risk-tolerant types who are pooled at B' also increases, since

$$-(1-p^H)dW_1^L - p^H dW_2^L = -(1-p^H)\left[1-\frac{V'(W_1^L)}{V'(W_2^L)}\right] dW_1^L > 0, \tag{7}$$

where the inequality is implied by equations (4) and (5), given $dW_1^L < 0$ and $W_2^L < W_1^L$.

Finally, the tangency condition (4) implies that the proposed change has no first-order effect on the expected utility that high-risk, low-risk tolerant types would obtain from the contract, since equation (5) yields

$$\begin{aligned}&(1-p^H)V'(W_1^L)dW_1^L + p^H V'(W_2^L)dW_2^L \\ &\quad = \left[(1-p^H)V'(W_1^L) - p^H V'(W_2^L)\frac{(1-p^L)U'(W_1^L)}{p^L U'(W_2^L)}\right] dW_1^L = 0,\end{aligned} \tag{8}$$

where the second equality follows from the tangency condition, which implies that the term within brackets vanishes. Hence, the self-selection constraint for the high-risk, low-risk tolerant types yields

$$V'(W^H)dW^H = (1-p^H)V'(W_1^L)dW_1^L + p^H V'(W_2^L)dW_2^L = 0\,, \tag{9}$$

where $W^H = W - p^H D$ denotes the wealth associated with H^*. Equation (9) implies that the proposed change in the contract underlying B' has no first-order effect on the profitability of the contract taken by high-risk, low-risk tolerant types, since $dW^H = 0$, while inequalities (6) and (7) indicate that profit earned on the partial pooling contract taken by low risks and high-risk, high risk-tolerant types increases. It follows that an incentive compatible pair of contractual offerings exists, one contract providing full coverage and attracting high-risk, low risk-tolerant types, and the other providing partial coverage and attracting low risks and high-risk, high risk-tolerant types, with profits earned on the latter more than sufficient to offset loses incurred on the former. As a result, the contracts underlying positions H^* and B' depicted in Figure 3 cannot be sustained as a Nash equilibrium.

Now suppose that high risks are homogeneous with respect to risk aversion and have utility function *V*, while some low risk share this utility function but others are more risk averse with utility function *U*. A case where the potential for profitable contracting arises is depicted in Figure 4. Again consider a differential change in the contract underlying point *B* such that the premium increases while holding constant the expected utility of the low risks. As before, this change increases the profit earned on these individuals and, because of the tangency condition (4), there is no first-order effect on the profitability of the high-risk contract. Once again the implication is that the configuration depicted in Figure 4 cannot be sustained as a Nash equilibrium.

Finally, when both risk classes are heterogeneous with respect to risk aversion, a greater variety of configurations with potentially profitable contracts is possible, but the arguments showing that these configurations cannot be sustained as Nash equilibria with free entry and exit remain the same. In each instance, the two-stage game theoretic model has no Nash equilibrium.

B.2. Nash Equilibrium in Three-Stage Screening Games

The Nash equilibrium of a three-stage game in which the uninformed insurers move first is a pooling contract. For simplicity, the following discussion of equilibrium configurations assumes that there are at most two degrees of risk aversion and that these are the same for applicants in both risk classes.

When the Rothschild-Stiglitz contracts are not sustainable and there are two types of high-risk applicants and one type of low risk, the equilibrium is the breakeven pooling contract most preferred by low-risk types, depicted as point *G* in Figure 5. This case most closely resembles the original Rothschild-Stiglitz environment analyzed by Hellwig. With one type of high-risk applicant and two types of low risks, the high risks pool with the low-risk, low risk-tolerant types at the breakeven pooling contract most preferred by the latter, while the low-risk, high risk-tolerant types receive fair insurance with a deductible set either by the preferences of the low-risk, low risk-tolerant types, as depicted in Figure 6, or by the preferences of high risks, whichever results in the higher deductible.[11] Finally, with heterogeneity in both risk classes, high risks pool with low-risk, low risk-tolerant types at the breakeven pooling contract most preferred by the latter, while low-risk, high risk-tolerant types receive fair insurance with a deductible set by the preferences of the pooled applicants that result in the highest deductible, as depicted in Figure 7.

4. Related Studies

In the model investigated by Wambach (2000), an insurance applicant's initial wealth is assumed to be hidden knowledge in addition to the risk of loss. If applicants do not exhibit constant absolute risk aversion, then unobservable differences in wealth give rise to unobservable differences in risk aversion. This, in turn, gives rise to the possibility of double-crossing indifference curves and self-selection contracts that earn positive profit. Since the ultimate cause of profitable contracts is the hidden knowledge of risk aversion, the analysis of the preceding section applies equally to this model, implying that the profitable contracting proposed as an equilibrium cannot be sustained under competitive conditions.

Sonnenholzner and Wambach (2004) introduce a temporal aspect to insurance contracting by assuming that the insurance premium is paid in the period preceding the one in which the loss may occur, and also incorporate a moral hazard whereby applicants can undertake an unobservable effort in self-protection that reduces the probability of loss. Self-protection effort may be high or low, and applicants differ with respect their unobservable rates of time preference, ρ. An applicant's expected utility is

$$U(W-P)+\frac{1}{1+\rho}\left\{[1-p(e)]U(W)+p(e)U(W-D+I)\right\}-c(e) \qquad (10)$$

when paying a premium P for insurance coverage I and bearing the cost $c(e)$ for self-protection effort e. Thus, in this model, risk type is private information that depends on the hidden action taken with respect to self-protection.

Given the initial wealth W and the potential loss D, an applicant's optimal choice of effort depends only on the applicant's rate of time preference and the insurance indemnity, while the applicant's willingness to pay for coverage depends on the rate of time preference and the choice of effort. Sonnenholzner and Wambach show that, under appropriate assumptions, for each value of ρ there exists a critical value for the indemnity such that high effort is exerted at indemnity levels less than or equal to the critical value and low effort is exerted at levels greater than the critical value.

One case in which profitable self-selection contracting arises has impatient types with high rate of time preference ρ^H exerting low self-protection effort, receiving full and fair insurance, and being indifferent between their contract and the contract take by patient types, who have a low rate of time preference $\rho^L < \rho^H$.[12] Patient types are only partially covered and they exert high effort, but incentive compatibility precludes their paying a fair premium and their contract earns positive profit.

To see that these contracts cannot be sustained as an equilibrium, consider the effect of ceteris paribus reductions in the two premiums such that

$$U'(W - P^L)dP^L = U'(W - P^H)dP^H\,, \tag{11}$$

thereby maintaining the self-selection constraint given utility function (10). Since the indemnities are unchanged, effort choices are unaffected and expected utilities increase. Let λ denote the proportion of impatient types, and let π^L denote the positive profit earned on the contract taken by patient types. The new contract for impatient types is unprofitable, but together the new contracts earn positive profit if the per-applicant reduction in premiums is less than the per-applicant profit of the original contracts, so that

$$(1-\lambda)\pi^{L} > -[(1-\lambda)dP^{L} + \lambda dP^{H}]. \qquad (12)$$

Combining conditions (11) and (12) yields an upper bound on the reduction in the premium of the impatient types,

$$\pi^{L} \Big/ \left[\frac{\lambda}{1-\lambda} + \frac{U'(W-P^{H})}{U'(W-P^{L})} \right] > -dP^{H}. \qquad (13)$$

The left-hand side of this inequality provides a positive upper bound for $-dP^{H}$, implying the existence of a pair of incentive compatible and jointly profitable contracts capable of attracting both types of applicants away from their original contracts. It follows that the original contracts cannot be sustained as an equilibrium under conditions of competitive contracting.

5. Conclusions

The possibility of jointly profitable self-selection contracts surviving as Nash equilibria in competitive insurance markets has been advanced in several studies that modify the adverse selection model of Rothschild and Stiglitz by introducing a second dimension of hidden knowledge. In three instances, the possibility of profitable equilibrium contracting arises because the indifference curves of the two risk types cross twice, in violation of the Spence-Mirrlees single-crossing property (in the fourth instance the possibility is attributable to the unobservable choice among discrete levels of a self-protection effort). In each instance, a profitable defection attracts low-risk (or patient) types away from the self-selection pair identified by Rothschild and Stiglitz, and any contract with a lower premium and comparable deductible would attract all applicants and earn negative profit.

While profitable contractual pairs can be sustained as equilibria in imperfectly competitive markets, they cannot survive under competitive conditions of free entry and exit with respect to insurers and the contracts they offer. There always exists a pair of incentive compatible defecting contracts that attract applicants away from the proposed equilibrium pair and jointly earn positive profit, with profit earned on one contract offsetting losses earned on the other. Hence, a jointly profitable pair of self-selection contracts in which one contract breaks even cannot survive in a competitive environment. Since neither cross-subsidized nor profitable separating contracts can survive as a Nash equilibrium for the two-stage model of insurance contracting, the three-stage model suited to insurance markets predicts an equilibrium with pooling.

Footnotes

[1] This configuration obtains in one case of the model examined by Sonnenholzner and Wambach (2004), when impatient types undertake low self-protection effort and patient types undertake high effort. In a second case, all applicants undertake the same level of self-protection, and so are equally risky but, as they differ with respect to their rates of time preference, they differ with respect to their willingness to pay for insurance coverage. In this case a proposed equilibrium provides impatient types with full and fair insurance and patient types with a profitable deductible contract. As shown in section 4, these configurations are vulnerable to a profitable, cross-subsidized pair of defecting contracts and cannot be sustained as Nash equilibria.

[2] Wilson (1977) and Riley (1979) have proposed non-Nash equilibrium concepts to resolve the nonexistence problem in the Rothschild-Stiglitz model. In Wilson's anticipatory equilibrium, insurers foresee the exit of contracts rendered unprofitable by a defection and are deterred from entering contracts that would then become unprofitable. In Riley's reactive equilibrium, insurers foresee the entry of contracts made profitable by a defection and are deterred from entering contracts that would become unprofitable as a result of such entry. Since it is difficult to rationalize the asymmetric foresight assumed in these equilibrium concepts, the analysis in this paper focuses exclusively on myopic, Nash behavior.

[3] Engers (1987) and Mas-Colell et al. (1995, p. 465) acknowledge that pooling contracts are not the only defections that can overturn the Rothschild-Stiglitz contracts, and recognize the greater threat posed by cross-subsidized pairs of incentive compatible contracts.

[4] The first-best pooling contract breaks even while providing full insurance to all applicants.

[5] Note that as λ increases, point *F* in Figure 1 moves down the 45-degree line. By totally differentiating equations (1) and (2), it is straightforward to show that the slope of locus *FA* decreases in absolute value as λ increases. When λ is greater than or equal to λ^*, the locus no longer rises above the low-risk indifference curve passing through point *A*, and the Rothschild-Stiglitz contracts constitute the Nash equilibrium for the two-stage screening game.

[6] Villeneuve (1997) provides an earlier English language version of Villeneuve (2003).

[7] Risk aversion implies that the high-risk acceptance set at H^* is strictly convex, so that point *B* must lie below the chord H^*A which, in turn, must lie below the locus *FA*.

[8] Given any cross-subsidized separating pair that earns nonnegative profit, there exists a profitable cream-skimming defection that attracts only the low risks, rendering the cross-subsidized pair unprofitable.

[9] Villeneuve also assumes that λ is high enough that no pooling contract can attract applicants from H^* and B while earning a positive profit.

[10] Debreu (1959, p. 41) identifies free entry, "i.e., no institutional or other barrier to entry," with additivity. If production sets are convex and own the zero vector, then additivity implies constant returns to scale. Under the free entry and exit conditions elaborated by Baumol et al. (1982) and associated with "contestable markets," investment in an entry fee earns a normal rate of return and is fully recoverable upon exit. In the atemporal Rothschild-Stiglitz model of insurance contracting, firms have no loading costs and the intertemporal rate of return is normalized to zero.

[11] Point F' in Figure 6, and in the subsequent Figure 7, represents the first-best pooling contract for high risks and low-risk, low risk-tolerant types.

[12] As noted earlier, Sonnenholzner and Wambach (2004) identify another case in which profitable self-selection contracting arises, but applicants are equally risky. However, the contractual configuration for case is qualitatively the same as this one, so the argument that follows applies equally to both cases, and shows that the proposed contractual configurations cannot be sustained as equilibria under competitive contracting.

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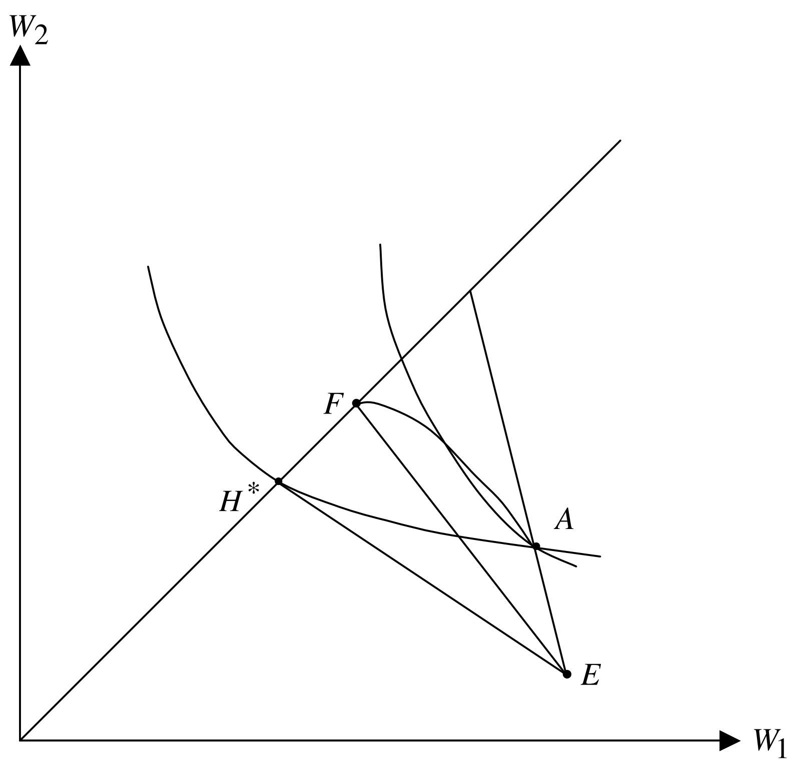

Figure 1

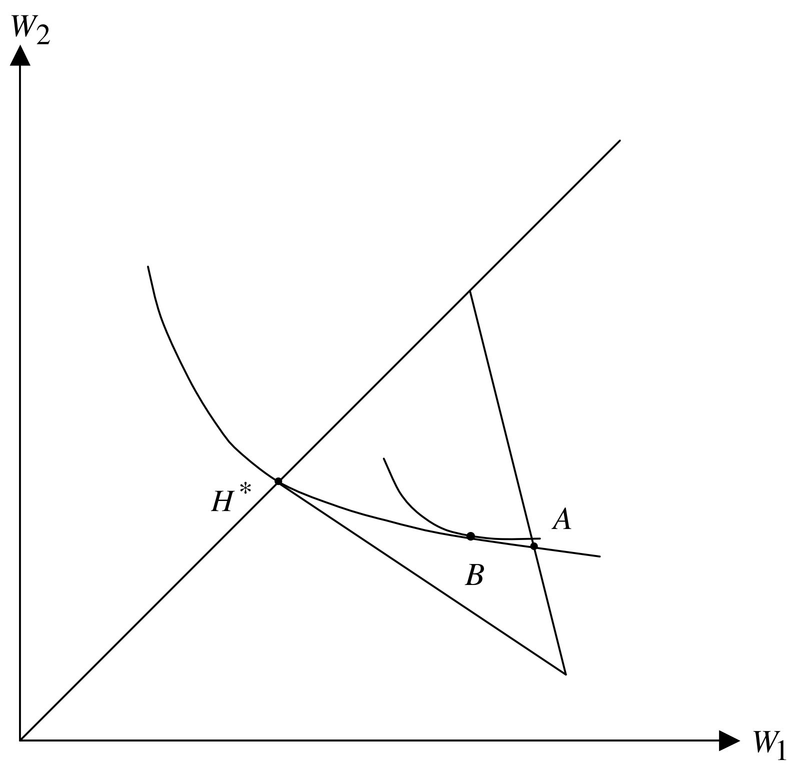

Figure 2

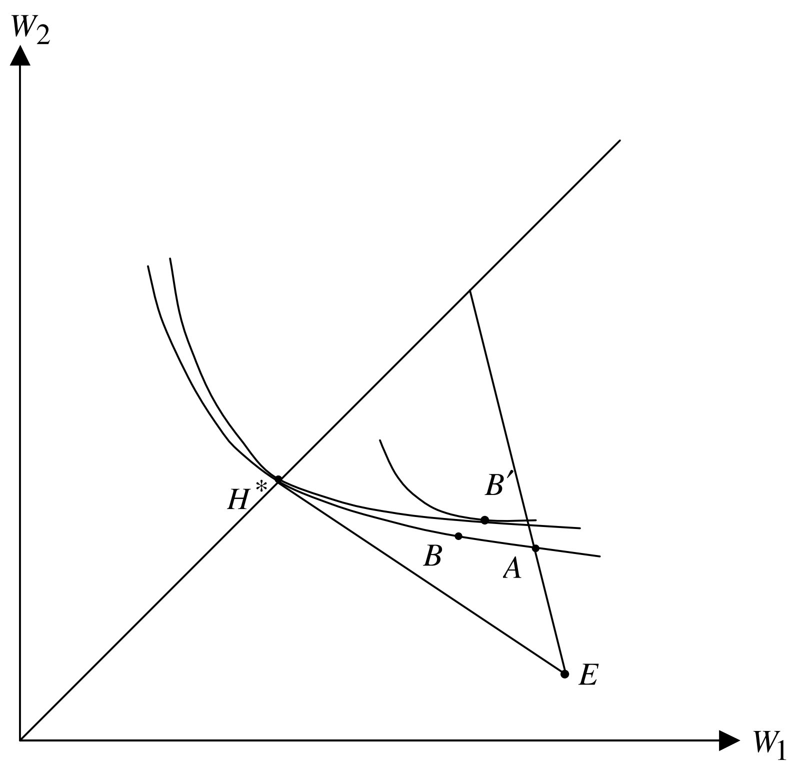

Figure 3

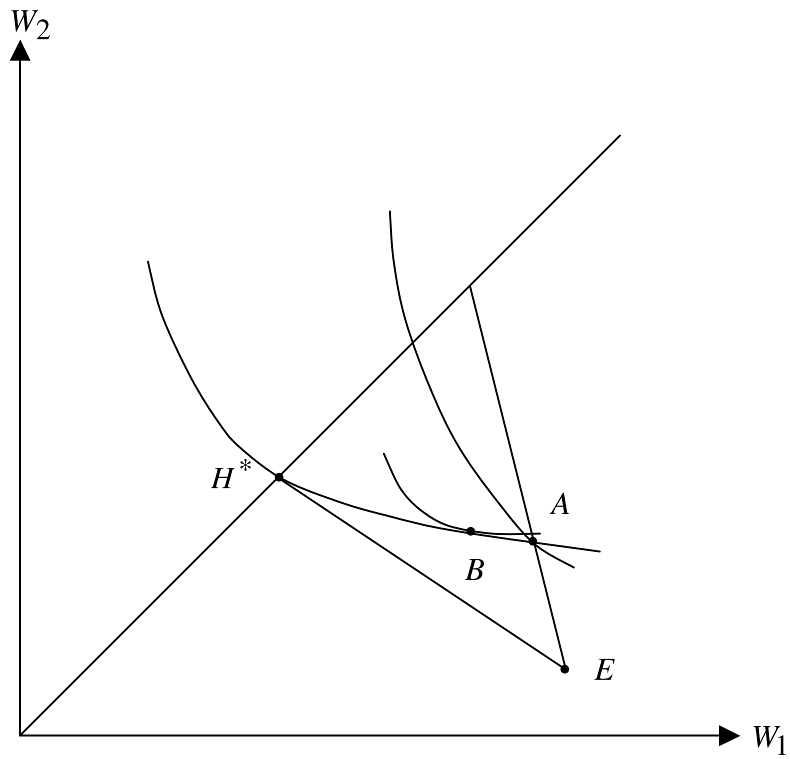

Figure 4

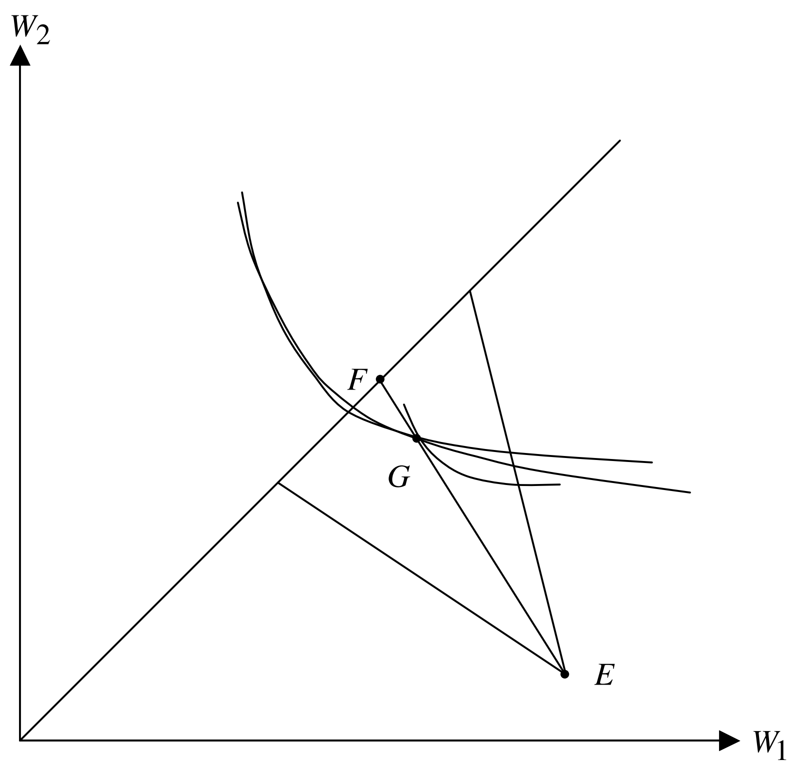

Figure 5

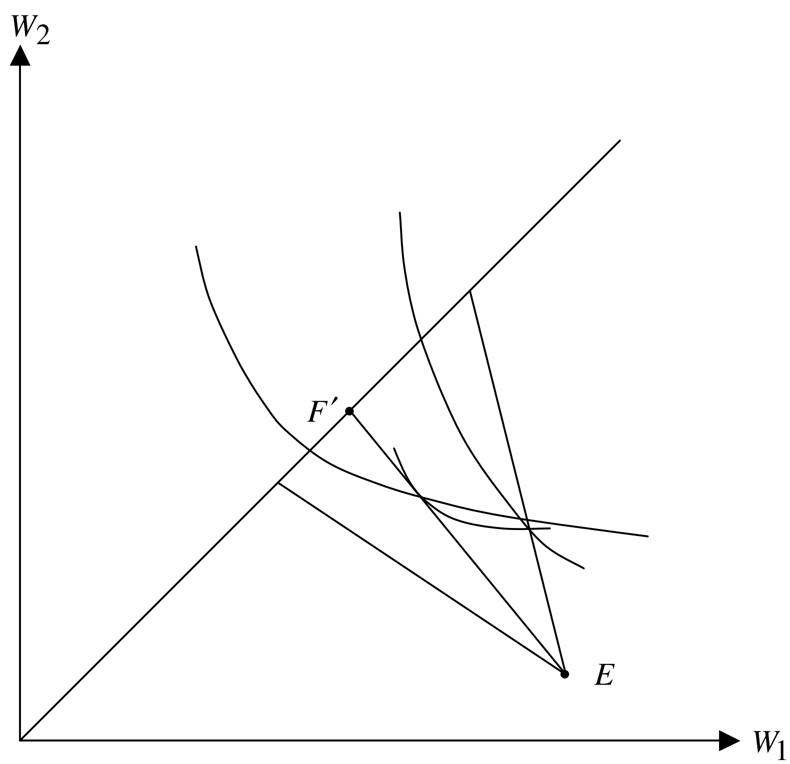

Figure 6

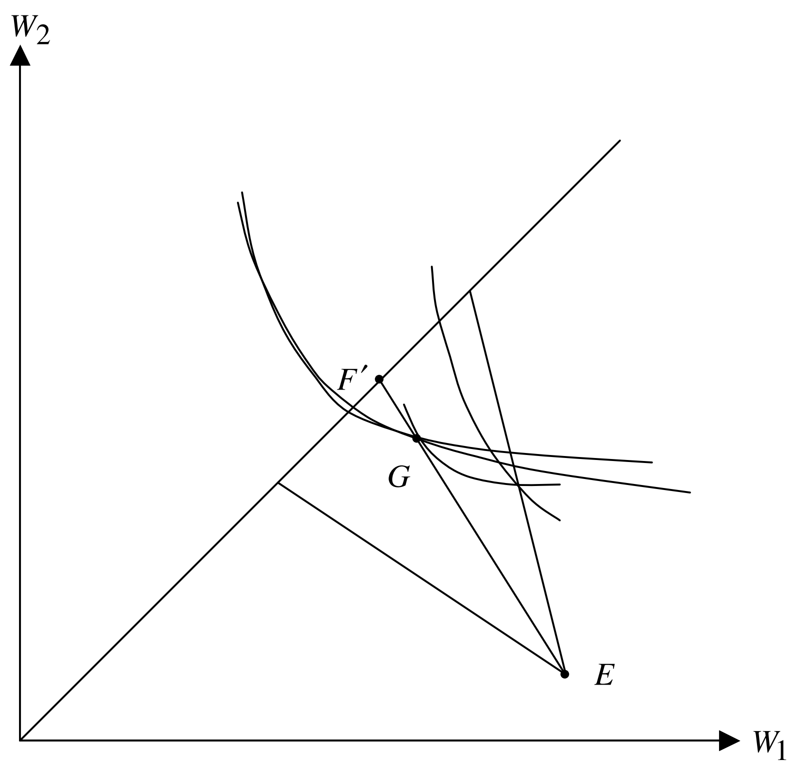

Figure 7